

Risk, Return and Rebalancing

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May 2005

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Abstract

Investors and fiduciaries would like to know the probable consequences of portfolio management elections prior to their implementation. This is especially the case for decisions to incur voluntarily extra costs and taxes by electing to rebalance the portfolio to its asset allocation targets. The literature discussing rebalancing is extensive. Various authors claim that rebalancing elections can control portfolio risk, enhance returns, or both. However, there are few studies examining the economic consequences of rebalance elections for well-diversified portfolios operating with realistic costs and differential tax rates for ordinary and capital gains income. Even fewer studies consider rebalancing elections under both accumulation and decumulation regimes. This paper, following a review of the literature, examines the output of a simulation model designed to test the marginal effects of rebalance elections on terminal wealth, aggregate consumption possibilities, and downside risk measures. The paper concludes that if terminal wealth has value because of bequest preferences or remainder interest considerations, the investor will select rebalance strategies to augment the utility of final dollar values. If, however, the investor does not have these preferences, the unspent money may merely represent lost consumption opportunities.

Modern Principles of Finance

One of the fundamental principles of modern finance is that an investment's risk and return can be best evaluated within the portfolio context. A collection of "safe" assets or "good" stocks that seem to be suitable investments when evaluated in isolation may, when combined, form a suboptimal portfolio. By contrast, a prudently diversified portfolio consists of investments that exhibit differing economic, accounting and statistical characteristics. When the investor fails to diversify, he or she runs the risk of having a collection of investments that move in lockstep. Although results may be very pleasant when investment returns couple and move towards the up side, the investor may suffer a catastrophic loss if returns couple and move downwards. The investor pursuing the "interior decorator" approach to portfolio construction seeks to purchase only those securities that fit well with his or her current tastes and preferences. However, these types of 'focused' or 'disciplined' approaches may create portfolios that generate extreme returns (positive or negative) under certain market conditions.

Diversification has been called the "antidote to uncertainty" because it reduces the dispersion of future investment returns. Future returns are uncertain because the investor does not know what they will be; nor, can the investor control their timing, sequence or magnitude. Risk (defined as the dispersion of a random variable about the mean of its probability density function), however, can be measured and managed so that the investment process is more likely to generate return sequences suitable to an investor's goals and objectives. Changing the shape of the density function through diversification is, on its face, merely a variant of the old maxim "don't put all your eggs in one basket." Diversification is, nevertheless, sometimes criticized because it fails either to maximize safety or to maximize returns;¹

¹ Interestingly, a diversified portfolio that truncates returns in any single period may, in fact, produce the greatest amount of ending wealth over a multiperiod planning horizon. This is because compound return (which determines the portfolio's terminal wealth) is approximated, in lognormal distributions, by the following equation:

Compound return = average return - $\frac{1}{2}$ (variance).

rather, diversification enhances the probability of a successful financial outcome over a broad spectrum of possible future economies.

A commonly used technique for achieving a well-diversified portfolio (i.e., a portfolio with appropriate risk/reward characteristics) is asset allocation. Asset allocation defines an appropriate opportunity set of investments (i.e., stocks, bonds, cash) and selects the proportion or weighting of each investment within the portfolio.² The selection and weighting decisions are a function of the investment's contribution to the risk and return characteristics of the portfolio—that is to say, investments are best evaluated in the portfolio context rather than in isolation.³ Effective portfolio design is, in large measure, a function of asset allocation guidelines.⁴ Indeed, a portfolio may be defined as a grouping of assets; and, portfolio management may be defined as the set of tools and techniques used to make them evolve in such a way that the investor's economic objectives are reached while respecting the investor's constraints, preferences and risk guidelines.⁵ Specifically, the portfolio's asset allocation should generate a return distribution that is well synchronized with the investor's risk tolerances as well as his or her accumulation and spending objectives.⁶

Variance, which is the dispersion of the density function about its mean, subtracts value from the portfolio; and, by mitigating variance (or, standard deviation which is the square root of variance), diversification makes the wealth-building process more certain. Strategies designed to maximize return over single time periods may be very risky if they ignore risk in their pursuit of high returns.

² Usually subject to one or more constraints such as no leverage (borrowing, speculative use of derivatives, or purchase of investments on margin); no negative positions in any investment (short sales); and, the sum of the investment proportions must equal 100%.

³ There are a variety of algorithms used to select investments from the opportunity set. Generally, however, combinations of investments with return sequences that are not strongly correlated are most efficient for controlling portfolio risk. If risk is defined as the standard deviation of returns (SD), expected or forecasted return is signified by E(R), and correlation is signified by the Greek letter rho (ρ), then adding a new investment to a portfolio is beneficial if:

$$\frac{E(R_{new}) - R_{riskfree}}{SD_{new}} > \left[\frac{E(R_{portfolio}) - R_{riskfree}}{SD_{portfolio}} \right] \rho_{new, portfolio}$$

The inequality sign creates an investment barrier or hurdle that is high when the new investment is strongly correlated to the portfolio, and low when the new investment is weakly correlated to the portfolio. Efficient portfolios don't like 'good' investments, they like 'uncorrelated' investments.

⁴ This essay considers neither financial engineering approaches to portfolio design (i.e., use of hedging techniques through derivative securities for risk control) nor actuarial approaches (i.e., use of annuities).

⁵ Amenc, Noel & Le Sourd, Veronique, Portfolio Theory and Performance Analysis (Wiley Finance Series, 2003), p. 6.

⁶ An important distinction must be made between the investor's "desired return" and the "required return." Investors who desire returns in excess of the returns necessary to achieve their financial objectives may wish to employ active management in the attempt to beat the market (or, more aptly, a well-specified return bogey). Care should be taken to select skilled investment managers offering a reasonable chance of adding value. That is to say, the investor must be highly confident that he or she possesses the ability to identify skilled managers who are likely to perform well over forthcoming economic environments. This endeavor is risky, however, because a failure to beat the bogey may mean that the investor cannot meet critical financial objectives. Thus, there is a risk/return tradeoff implicit in the choice of investment management strategy. Active investment management may provide additional funds (more money is better than less) at the risk of failing to achieve the required return; passive management secures market-based returns and broadly diversified portfolios (i.e., avoids making risky bets) but truncates the investor's ability to earn excess wealth. When making the choice between passive and active investment management it is vital to determine if your investment goal can be defined as an attempt to solve an "intertemporal cash flow problem" or as an attempt to "beat the market." Solving an intertemporal cash flow problem is moving money from the future to the present (i.e. borrowing to buy a house) or moving money from the present to the future (i.e., investing for retirement). The attempt to beat the market involves a speculative dimension that may or may not be justified in terms of the underlying investment objectives.

Portfolio Management v. Investment Management

Typically, asset allocation guidelines fall into either a “fixed mix” structure⁷ in which the portfolio maintains constant exposures to the risks and returns of the selected asset classes in both bull and bear market conditions; or, into a “tactical allocation” structure in which minimum and maximum levels of exposure are set for each asset class. In the latter case, the expectation is that the investor will tilt the asset allocation weights according to market conditions, with the magnitude of the tilt conditioned upon his or her market forecasts and upon his or her confidence in the forecast’s accuracy.⁸ The fixed mix sets specific asset weighting targets while the tactical allocation structure sets ranges over which asset weightings are allowed to vary.⁹

One interesting form of tactical asset allocation that is not based on market prognostications is the “insured portfolio” approach (commonly known as “portfolio insurance”).¹⁰ Portfolio insurance sets a floor below which the investor’s wealth should not penetrate. Commonly, this floor is 20% or 30% below the initial portfolio value. In addition to the floor value, the tactical allocation range sets a ‘multiplier’ value. When the investor’s wealth is above the floor value, the multiplier is applied to the surplus to determine the amount of stock market exposure. For example, a \$1 million portfolio may have a floor value of \$800 thousand and a multiplier of 4. The surplus is $(\$1,000,000 - \$800,000)$ or $\$200,000 \times 4 = \$800,000$. Thus, at time zero, the portfolio is allocated \$800 thousand to risky investments (stocks) and \$200 thousand to risk free investments (T-Bills). If the portfolio value moves up over time (i.e. the surplus increases) the proportion of the portfolio subject to stock market exposure also will increase—a bigger cushion allows the investor to take more risk; if the portfolio value decreases, the multiplier will force the investor to unwind the stock exposure in favor of putting the money into T-Bills. Under a severe bear market condition (i.e., the portfolio loses 20% of its initial dollar value), the remaining \$800 thousand is invested in T-Bills. The insured portfolio approach is “active” in the sense that it responds to market conditions, but “passive” in the sense that its response is based on actual rather than predicted economic circumstances. Stock price forecasting is not a part of an insured portfolio management system.

Likewise, the distinction between “desired risk” and “required risk” is also of great importance. Portfolios that take only the required amount of risk have the expectation of producing a tighter distribution (range) of future results. Active management, because it deviates from the consensus position of the market, may expose the investor to certain risks that are undesired and, perhaps, hidden or unintended.

⁷ Also known as “constant mix.”

⁸ Note that the fixed mix is static but “objective” while the tactical allocation is dynamic but “subjective.” Some commentators believe that it is better to keep the portfolio approach “objective,” and to change the asset allocation guidelines only as a response to changes in the personal circumstances or financial goals of the investor. The danger of basing portfolio management on “subjective” criteria is an overreaction to transitory market conditions—investing too aggressively following an up market trend (ignoring risk) and investing too conservatively following a down market trend (avoiding risk). The danger of a fixed mix strategy is that it fails to take full advantage of the current information set which may contain variables with predictive content for the investor.

⁹ The fixed-mix allocation is sometimes termed long-term or ‘strategic’ asset allocation. By contrast, tactical asset allocation is a response to market predictions for the forthcoming period: “Tactical asset allocation (TAA) ... involves making short-term adjustments to asset class weights based on short-term predictions of relative performance among asset classes....TAA creates active risk....In exchange for active risk, the manager using TAA expects to earn positive active returns that sufficiently reward the investor for the risk taken.” Pinto, Jerald E., & McLeavey, Dennis, W., “Strategic Asset Allocation Concepts,” CFA Institute ([CFA Level III Readings in Portfolio Management](#), 2005), p.283.

¹⁰ Perold, Andre F., & Sharpe, William F., “Dynamic Strategies for Asset Allocation,” [Financial Analysts Journal](#) (January-February, 1988), pp. 16-27. The intellectual roots of this strategy can be traced to Fischer Black and others. See, for example, Black, Fischer, “Individual Investment and Consumption under Uncertainty,” [Portfolio Insurance, A Guide to Dynamic Hedging](#), edited by Donald L. Luskin (John Wiley & Sons, 1988), pp. 207-225.

Once the asset allocation targets are set, the investor may choose to employ active management (forecasting and market timing strategies), passive management (broadly diversified, market-based returns) or a combination of each.¹¹ Irrespective of the investment management approach, there remains the question of how the aggregate portfolio should be managed. A passive portfolio management approach is a buy-and-hold or 'drift' approach;¹² an active portfolio management approach can take a variety of forms including tactical asset allocation (tilting the portfolio's investments within asset allocation ranges) or rebalancing to the fixed mix targets. At first blush, it seems that fixed-mix rebalancing, portfolio insurance, and tactical asset allocation are first cousins because each requires the investor to take active portfolio management steps. However, in practice, tactical asset allocation is a market trend anticipating strategy. By contrast, the portfolio insurance approach to asset management is a trend following strategy. Tactical asset allocation takes a prospective view of the market; portfolio insurance takes a retrospective view of the market; and rebalancing to a fixed mix does not require a market viewpoint.

Active Portfolio Management: Rebalancing

This paper explores certain topics relating to portfolio management as opposed to investment management. For most, but not all, investors an active approach to portfolio management is preferred. Stated differently, even a portfolio that has all its investment positions passively managed (e.g., a portfolio of index funds), cannot be neglected or ignored.¹³ Its cautionary message is that investors should select investment advisors that provide investment management approaches or portfolio management structures that are sufficiently flexible to suit to their needs, circumstances, and economic objectives. Although the cautionary message sounds trite, a moment's reflection should be sufficient to raise a number of interesting questions:

1. Should portfolio management approaches used for a taxable portfolio be similar to those used for a tax-favored portfolio such as a qualified retirement plan?
2. How does a fixed-mix approach compare with a buy-and-hold approach or with other active portfolio management strategies?
3. Can a fixed-mix approach be dynamic? Can it be used in a portfolio with ongoing cash flows (contributions and withdrawals), or in a portfolio designed to fund dynamically changing liabilities such as pension obligations, endowment funding requirements, or personal retirement income needs in the face of inflation?
4. How and when should asset allocation guidelines be changed?
5. As the portfolio evolves, what guidelines should be used to control the portfolio's exposures to the risks and rewards of the asset allocation guidelines?

¹¹ Treynor, Jack L. & Black, Fischer "How to Use Security Analysis to Improve Portfolio Selection," Journal of Business (January, 1973), pp. 66-86.

¹² A seminal article written by Robert Kirby in 1984 discusses a passive portfolio approach known as a "coffee can" portfolio. Initially, specific stocks are actively selected based on the investor's determination of a company's long-term prospects. However, once the selection has occurred, the stock certificates are put into a "coffee-can" which is placed out of sight for a lengthy period of time. Kirby speculates that active investment management is unlikely to add value once the portfolio is implemented. The investor either has the ability to select stocks for the long haul, or he doesn't. If such ability is present, then why incur trading costs by buying and selling during the interim periods? Kirby, Robert G. "The Coffee Can Portfolio," The Journal of Portfolio Management (Fall, 1984), pp. 76-80.

¹³ Benign neglect is often not benign. However, for certain investors a passive or buy-and-hold approach to portfolio management is prudent and suitable. This is a topic discussed below.

6. How does an investor know what portfolio management structure is best for them?

There is a growing body of academic literature on the topic of portfolio management. Much of the literature focuses on techniques for improving the risk/return tradeoffs faced by non-taxable investors with wealth accumulation objectives.¹⁴ Some recent research discusses taxable investors as well as investors operating portfolios subject to cash flows. Many scholarly works contrast an actively managed portfolio to a portfolio managed under a buy-and-hold strategy. Although somewhat of an oversimplification, a survey of the extent literature reveals that research approaches generally fall into two categories:

1. An historical approach to the subject; or,
2. A mathematical approach to the subject.

Some papers employing an historical approach are subject to the flaw in statistical analysis known as “data mining.” Although the term “data mining” covers a wide range of abuses in sound research methodology,¹⁵ in this case it refers primarily to basing a conclusion on a sample of data that may not be representative of the true, but largely unknown, population of both past and future results. Statements such as “based on historical back testing, you should rebalance the portfolio to its asset allocation targets on a semi-annual basis in order to generate the greatest amount of ending wealth,” although possibly correct in a narrowly defined sense, are merely ad hoc rules that either (1) do not apply to the investor (because of interim cash flows or the need for dynamic liability matching); (2) are sensitive to the beginning and ending dates of the historical period under evaluation; or (3) merely reflect past occurrences without exhibiting predictive force under future economic conditions.¹⁶ By contrast, many useful insights flow from studies using more mathematical approaches. However, the benefits of such approaches are often limited in practice by the demands imposed by intensive tax calculations, transaction costs, portfolio software tracking limitations, limitations in investment advisor skill sets, custodian capabilities, and other real world constraints.

Rebalancing is a type of active portfolio management strategy employed, depending on the study under consideration, either to enhance returns, control risk or both. A simple example illustrates both the concept and mechanics of rebalancing. Assume a portfolio that, at time zero, places \$1,000 into each of three investments. This is comparable to stating that the portfolio has a one-third allocation to each asset. Over the forthcoming period, investment A earns a return of 10% (\$1,100); investment B earns a return of -3% (\$970) and investment C earns a return of 5% (\$1,050). At time period one, the aggregate portfolio has a value of \$3,120. In order to maintain the one-third allocation, the investor sells \$60 of investment A and \$10 of investment C. The \$70 in sales proceeds is reallocated to investment B. Each rebalanced investment position has a value of \$1,040, which is exactly one-third of the portfolio's total period one value.

This simple example provides several interesting insights. The rebalance decision forces the investor to sell high and buy low. This is one of the hallmarks of a contrarian market strategy.¹⁷ This sounds like a good thing because the opposite strategy of buying high and selling low seems to be a

¹⁴ Maximize terminal wealth subject to certain investment constraints.

¹⁵ See, for example, Black, Fisher, “Estimating Financial Return,” *Financial Analysts Journal* (September/October, 1993).

¹⁶ A good example of simplistic, rule-of-thumb maxims is found in Trone, Donald B., Allbright, William R., & Taylor, Philip R., *The Management of Investment Decisions*, McGraw-Hill (New York, 1996) p. 215: “An optimal limit would require readjustments twice a year on average—more than twice a year and the benefits may be eroded by transaction costs....”

¹⁷ A strict contrarian strategy, however, is a function of valuation estimation as much as it is a function of trading against the crowd. There is no forecasting in a mechanical portfolio rebalancing system. The strict contrarian investor hopes that he or she will not be “bagged” by momentum traders because of a misestimation of the intrinsic or justified value of the securities.

recipe for losing wealth. Investors pursuing a fixed mix rebalancing strategy are natural counterparties to investors pursuing an insured portfolio management strategy. The fixed mix investor, in the process of portfolio rebalancing, sells insurance. Under an insured portfolio management strategy, the investor moves out of risky assets when they decline in value and moves into risk-free Treasuries. Portfolio rebalancing, by contrast, demands that the investor buy into weakness and sell into strength because money is moved out of the relative winners and into the relative losers. Thus, when seen in terms of buying or selling insurance, it seems as if rebalancing should add value to returns (insurance has a premium cost and if you sell insurance you expect to receive a fair price for it). Paradoxically, however, the purchase of insurance mitigates risk while the sale of insurance increases risk (technically, the risk is not increased but is transferred from the buyer to the seller for consideration received). The failure of the insured portfolio approach under conditions of extreme market volatility is, however, an issue that lies outside of this discussion.¹⁸

Empirical Studies

Moving from the simple hypothetical example into the study of historical data provides a variety of interesting and sometimes contradictory results.¹⁹ As an illustration, we cite two studies by Craig Israelsen at the University of Missouri.²⁰ The first, appearing in 2001, considers the historical returns of portfolios invested equally in US large company stocks, US small company stocks, and International large company stocks. Israelsen evaluates results over several time periods and compares a drifting-mix passive portfolio management approach to a constant mix (annually rebalanced) active portfolio management approach. The initial value of each portfolio is \$1,000. The end-of-period results appear in the following table:

	1970-2000	1991-2000	1996-2000
Dollar Value of Drifting Mix	\$42,961	\$4,084	\$1,803
Dollar Value of Fixed Mix	\$49,496	\$3,974	\$1,795
Dollar benefit of annual rebalancing	\$6,535	(\$110)	(\$8)
Risk reduction benefit of rebalancing as measured by % reduction in Standard Deviation of returns	18.5% less risk	23.5% less risk	17.7% less risk

These results are noteworthy because they suggest that the return enhancing benefits of portfolio rebalancing are sensitive to the time period under consideration. However, the data also suggest that annual rebalancing reduces portfolio risk making the final financial outcome more certain. Israelsen

¹⁸ A more comprehensive comparison of fixed-mix vs. insured portfolio management structures is found in Farrell, James L., Portfolio Management Theory and Application Second Edition (Irwin McGraw-Hill, 1997), pp. 291-297. Farrell points out that "any strategy giving a convex payoff pattern is representative of the purchase of portfolio insurance, but strategies such as the constant-mix that give concave patterns represent the sale of insurance." He concludes: "convex strategies are inherently volatile and demand significant liquidity because of the trading demanded."

¹⁹ An interesting example is provided in: Horvitz, Jeffrey E., "The Implications of Rebalancing the Investment Portfolio for the Taxable Investor," The Journal of Wealth Management (Fall, 2002), p. 51: "In 1987 the stock market fell dramatically in a day. An investor who rebalanced quickly reaped substantial benefits. In 1929 the stock market also fell dramatically, but an investor who rebalanced immediately (and thereafter) probably was soon eating at soup kitchens, as the market plummeted in subsequent years."

²⁰ Israelsen, Craig, "Rebalancing Acts," Financial Planning (June, 2001) pp. 59-62; and "Rebalance of Power," Financial Planning (April, 2002) pp. 102-106.

concludes: “Asset allocation can serve the important function of dampening the downside risk without unduly penalizing return.”

His second study covers only the single twenty-five year period from 1977 through 2001. However, the study incorporates investment costs by comparing drifting-mix and fixed-mix results achieved by investors in three mutual funds (Vanguard 500 index fund, Vanguard US Small Cap index fund, and Scudder International Stock fund). For this study, the initial portfolios are each valued at \$3,000 instead of \$1,000; and all other assumptions are the same. In this case, rebalancing the portfolio to the fixed mix allocation generates 9% more money with 17.6% less volatility.²¹

Unfortunately, however, the conclusions of Israelsen’s studies have only limited application to portfolio management decision making. This is true for at least three reasons:

1. Most portfolios are not strictly accumulation vehicles but operate under conditions of cash inflows and outflows. This means that terminal portfolio wealth (as well as, perhaps, interim consumption financed from the portfolio) is path dependent. Given the fact that a precise repetition of the sequence of past returns is highly improbable, past behavior may not be an indication of future results.
2. Empirical results derived from historical rebalance studies depend greatly on the selection of the number of assets within the portfolio, the weighting (asset allocation) of each asset, and on their statistical relationships. In portfolios where the covariance matrix exhibits cross product entries that are negative or close to zero, compound multiperiod growth becomes an *increasing* function of the standard deviation of each asset. This greatly increases the benefits of rebalancing. The opposite may be true for portfolio formed from assets exhibiting individual standard deviations or higher values for the correlation statistic.²²
3. Additionally, there is the problem of creating an equal playing field for risk. A portfolio that produces more return for less risk is said to dominate a portfolio that produces less return at greater risk. However, how do we compare two portfolios when one has both greater return and greater risk? Under certain conditions, the return-per-unit-of-risk for each portfolio²³ can

²¹ Israelsen’s findings (rebalanced portfolios produced money terminal wealth with less standard deviation—higher Sharpe reward to risk ratios) parallel the results reported by Ibbotson Associates for a two-asset class portfolio allocated 60% to large cap US stocks and 40% to intermediate term bonds over the twenty-five year period ending September 2002. www.ibbotson.com/content/kc_search_detail.asp?catalog=AskIbbotson.

²² The geometric growth (compound return over time) of a portfolio can be approximated by the following formula:

$$G \approx \sum_i X_i G_i + \frac{1}{2} \left(\sum_i X_i V_{ii} - \sum_{i,j} X_i X_j V_{ij} \right)$$

The first term on the right hand side of the equation says that the sum of individual asset weights [X_i] times their individual growth rates [G_i] contributes to the portfolio’s compound return. However, the second term on the right says that additional compound growth depends on the *difference* between asset variance [V_{ii}] and asset covariance [V_{ij}]. The presence of a ‘minus’ sign means that assets with negative covariance and high variance (risk) can contribute positively towards multiperiod compound returns (a negative times a negative equals a positive). This phenomenon is known as the “Diversification Bonus” that is available from scientific asset allocation. Unrebalanced portfolios are positive in the first term (wringing out maximum compound value from the highest return assets) but are negative with respect to the second term (because of the failure to take advantage of low or negative correlation). Over a sufficiently long period of time, the unrebalanced portfolio’s return is dominated by the return of the single high compound-return asset. However, if the average returns of the assets within the portfolio are relatively close, the rebalanced portfolio will always be superior to the the unrebalanced portfolio (i.e., the second term on the right will be the greater contributor to return over time). This phenomenon is known as the “Rebalancing Bonus.” Bernstein, William J., & Wilkinson, David, “Diversification, Rebalancing, and The Geometric Mean Frontier,” (November 24, 1997) <http://www.effisols.com/basics/rebal.pdf>.

²³ When return is defined as [(the excess return over the risk-free rate) ÷ (standard deviation of return)], the return-per-unit-of-risk is the well-known Sharpe Ratio named after Nobel Prize winner William Sharpe. Sharpe ratios are comparable only

be evaluated to determine which portfolio has the superior risk-adjusted return. But there is the expectation that if you hold a portfolio allocated between stocks and bonds under a drifting-mix passive management regime, it will eventually approach a portfolio that is 100% stocks given a sufficiently long planning horizon.²⁴ Not only is the risk of a drifting mix portfolio much greater, the expected risk increases over time as equities dominate the asset weightings. Reward to risk ratios may not be the best comparative measure under these circumstances. This problem is known as “equity drift” and it presents a difficult task to the researcher wishing to make an apples-to-apples comparison of a static portfolio and a dynamic portfolio over time.

There are a variety of ways to achieve “risk-calibration,” but the technical details are beyond the scope of this paper.²⁵

Rebalancing Strategies

Why would an investor be unwilling to rebalance a portfolio? There are several arguments against an active portfolio management process utilizing rebalancing transactions:

1. Rebalancing costs money: trading is not free and, in some instances, trading costs can be formidable (trade commissions, market impact, bid/ask spreads, surrender charges, redemption fees, unamortized front end loads, etc.);
2. Rebalancing may trigger tax liabilities for taxable portfolios: this is an especially acute problem for under diversified portfolios invested in only a few assets with a low tax basis; and,
3. Rebalancing can truncate returns as money is taken away from winners and put into losers.²⁶ This problem may be especially acute in trending markets.

when the investment periods are the same. That is to say, it is not possible to compare, on the same scale, variance over unequal holding periods.

²⁴ This is because the expected return from stocks is higher than the expected return from bonds or cash. However, the fact that the stock investment is riskier (has a greater dispersion in its returns) pushes the planning horizon out to amazingly long periods. Mark Rubinstein (University of California, Berkeley) has a most interesting proof of this fact. He asks: “how long must an investor be prepared to wait before the probability becomes high that an all-stock portfolio will outperform an all-bond portfolio?” Rubinstein develops the following theorem: Assume that all available assets collectively follow a stationary random walk in continuous time (with finite variance). Let X and Y be the values after elapsed time $t > 0$ from following two strategies (with equal initial total investment), each being the result of continuously rebalancing a portfolio to maintain constant proportions in the available assets. Then:

$$\text{prob}(X > Y) = N\left\{ \frac{(\mu_X - \mu_Y)\sqrt{t}}{[\sigma_X^2 - 2\rho\sigma_X\sigma_Y + \sigma_Y^2]^{1/2}} \right\}$$

where N is a joint standard lognormal probability distribution, $\mu_X t$ is the expected value of log (X), $\mu_Y t$ is the expected value of log (Y), $\sigma_X \sqrt{t}$ is the standard deviation of log (x), $\sigma_Y \sqrt{t}$ is the standard deviation of log (Y), and ρ is the correlation between log (X) and log (Y). Assuming, based on a reasonable sample of historical data, that stocks offer a 2.5% return premium over bonds, with the standard deviation of stocks equal to 18% and the standard deviation of bonds equal to 10% with a correlation of 0.4, in order to be 95% confident that an all stock portfolio will outperform an all bond portfolio requires a planning horizon of 123 years. This is a handy theorem to remember the next time you hear someone say that long-term portfolios should consist of 100% equity because stocks have outperformed bonds in every 10-year period. Rubinstein, Mark “Continuously Rebalanced Investment Strategies,” Journal of Portfolio Management (Fall, 1991).

²⁵ A common adjustment methodology involves simulation of returns to determine the average degree of equity drift. The starting equity weight of the fixed-mix portfolio is adjusted to this equity level so that risk exposures, on average, are comparable. See, for example, Buetow, Gerald W., Sellers, Ronald, Trotter, Donald, Hunt, Elaine & Whipple, W.A., “The Benefits of Rebalancing,” The Journal of Portfolio Management (Winter, 2002), p.26.

Rebalancing is, therefore, a process that balances transaction and tax costs against potential risk and return enhancements. If a portfolio calls for a 25% allocation to investments within a specific asset class, it would probably not be cost-effective to initiate rebalance transactions if the weighting of the asset drifts to 25.1%. The one-tenth of one percent differential will probably not have a marked effect on returns nor important consequences with respect to risk.²⁷ Therefore, a major argument for rebalancing is, in a nutshell, that it preserves diversification by adherence to the asset allocation guidelines. A major argument against rebalancing is that it costs money.

Several academic studies focus on the topic of rebalancing rules. In general, rebalancing rules prescribe both formulae under which the investor initiates rebalancing, and the extent of the rebalancing actions. A review of the literature indicates that, in general, the formulae initiating rebalancing actions fall into one of five categories:

1. Calendar based rules: rebalance daily, monthly, quarterly, yearly, etc.²⁸;
2. Drift from initial asset allocation proportions (i.e., if a 10% proportionate drift triggers a rebalance action, for a two asset class portfolio with a 60% stock allocation, the investor would initiate rebalance transactions if the stock grew to 66% of the portfolio's value or diminished to 54% ($\pm 10\%$ of 60%);²⁹
3. Fixed percentage drift (i.e., if an asset class drifts more than $\pm 10\%$ of total portfolio value, the investor takes rebalancing actions. For example, given a 60% allocation to stock, if stock value increases to more than 70% weighting or decreases to less than 50% portfolio weighting, the investor would take rebalance action. In this category, the investor can set differing upper and lower bounds for each asset class);³⁰
4. Standard Deviation criteria: rebalancing occurs when an asset's risk premium (return in excess of the risk-free rate) shows a marked increase or decrease over its historical mean;³¹

²⁶ An investor may have neither a well-defined financial goal (a generation-skipping trust's goal may simply be expressed in terms of growing wealth for a future generation), nor a well-defined planning horizon (e.g. a "dynasty" trust), nor a benchmark allocation calibrated to specific economic objectives and risk tolerances. Drifting away from the benchmark is, therefore, not a particular concern. Such an investor is said to be indifferent with respect to risk. An additional utility-based argument against rebalancing is discussed later in the paper.

²⁷ If however, the portfolio value is substantial, small increments translate into large dollar amounts.

²⁸ Arnott, Robert D., & Lovell, Robert M., "Rebalancing: Why? When? How Often?" *Journal of Investing* (Spring, 1993), p. 7, suggest that monthly rebalancing "...might add 41 basis points to the annual return of a 50/50 portfolio (9.16% versus 8.75%)." The gain is equal to increasing the allocation to equity by 11 percent. Yearly rebalancing added value that was almost equivalent to monthly rebalancing (9.02% versus 9.16%). Dennis, Patrick, Perfect, Steven B. & Snow, Karl N., "The Effects of Rebalancing on Size and Book-to-Market Ratio Portfolio Returns," *Financial Analysts Journal* (May-June 1995), pp. 47-57, note that optimal rebalancing for a long-only portfolio is two years.

²⁹ Stine, Bert & Lewis, John, "Guidelines for Rebalancing Passive-Investment Portfolios," *Journal of Financial Planning* (April, 1992), pp. 80-86.

³⁰ Evensky, Harold, "Rebalancing Act: A formula for revalancing to avoid style drift," *Financial Planning* (June 1996), pp. 170-171, 193).

³¹ See, for example, Arnott, Robert D., & Lovell, Robert M., "Monitoring and Rebalancing the Portfolio," *Managing Investment Portfolios: A Dynamic Process* edited by John L. Maginn & Donald L. Tuttle (Warren Gorham & Lamont, 3rd edition), Chapter 13. Standard Deviation approaches can also be based on asset volatility or heteroskedasticity which triggers rebalance actions based on extreme movements in asset values.

5. Other criteria: including continuous rebalancing using a derivatives portfolio overlay (futures contracts), probability-based rebalancing based on creating of value-added confidence intervals, and so forth.³²

Recommendations regarding the extent of rebalancing (as opposed to the frequency dictated by the above-listed formulae), generally fall into one of three categories:

1. Rebalance to the normal policy targets (i.e., for the fixed-mix portfolio rebalance to the asset allocation targets);
2. Rebalance to the boundary or threshold limit at which the transaction is triggered; or,
3. Rebalance to a point between the asset allocation target and the boundary.

Mathematical Studies

Hayne Leland at the University of California, Berkeley, authored a seminal mathematical study of rebalance tradeoffs.³³ Drawing upon earlier studies by George Constantinides, and B. Dumas & E. Luciano,³⁴ Leland's study argues that when rebalancing costs are proportional to the amount rebalanced,³⁵ the optimal strategy involves:

1. Placing a "no-trade" zone around the asset; and,
2. If the proportional weighting of the asset within the portfolio penetrates the no-trade barrier, rebalancing should be undertaken to restore the asset weighting to the nearest edge of the barrier rather than to the nominal asset allocation target.

Leland does not specify a particular utility function for the investor,³⁶ but develops a cost function for deviations from the targeted asset allocation. If the target allocation is well synchronized to the investor's

³² For a discussion of a derivatives overlay portfolio see: Buetow, et al., Supra. Probability-based rebalancing is the subject of a paper by Michaud, Robert O., and Michaud, Richard O., "Resampled Portfolio Rebalancing and Monitoring," available at www.newfrontieradvisors.com. This paper is an extension of research originally published in Michaud, Richard O., Efficient Asset Management (Oxford University Press) 1998. Rebalancing to a "peer group" consensus benchmark is discussed in Lowe, Stephen, "Rebalancing the Portfolio," Asset Allocation in a Changing World (AIMR 1998), pp. 117-125.

³³ Leland, Hayne E., "Optimal Asset Rebalancing In the Presence of Transaction Costs," Working Paper No. RPF-261 Walter A. Haas School of Business Research Program in Finance Working Paper Series (August, 1996). See also, Leland, Hayne E., "Optimal Portfolio Management with Transactions Costs and Capital Gains Taxes," Research Program in Finance Work Papers (University of California, Berkeley, 1999), Paper RPF 290.

³⁴ Constantinides, George, "Capital Market Equilibrium with Transactions Costs," Journal of Political Economy (1986), and Dumas, B., & Luciano, E., "An Exact Solution to a Dynamic Portfolio Choice Problem Under Transactions Costs," Journal of Finance (1991), pp. 577-596.

³⁵ Transactions costs can be fixed (i.e. irrespective of the amount bought and sold the investor has only a fixed charge) or proportional. If proportional costs are, for example, one percent, a trade involving \$1,000 of assets will cost \$10 while a trade involving \$1,000,000 of assets will cost \$10,000.

³⁶ Utility functions are the inverse of risk aversion functions. Risk aversion function graphs are usually increasing convex with positive first and second derivatives (the more risk undertaken, the more reward demanded; and the demand for reward increases at a rate faster than the risk assumed—because the pain of dollar lost is greater than the joy of a dollar gained). The inverse utility of wealth curve is usually increasing concave with a positive first and a negative second derivative—utility (satisfaction) with wealth increases as one earns an extra dollar but increases at a decreasing rate (a millionaire gains satisfaction by adding another dollar of wealth but his or her satisfaction is not as great as the pauper who adds a dollar to his or her wealth).

wealth accumulation and consumption objectives, deviation from the target will result in a measure of “disutility” to the extent that it makes the attainment of future economic goals more uncertain.

Leland’s model uses two asset classes (stocks = S and bonds = B) that follow log random walks (Brownian Motion).³⁷ Therefore, according to the fundamental law of the evolution of wealth, the instantaneous rate of change for the stock position equals:

$$\frac{dS(t)}{S} = \mu_S dt + \sigma_S dZ_S(t)$$

and, for the bond position, equals

$$\frac{dB(t)}{B} = \mu_B dt + \sigma_B dZ_B(t)$$

These equations state that the change in wealth follows a process dependent on three factors: the expected return (μ = mean or average return); the expected standard deviation of return (σ = standard deviation) and an adjustment to the expected standard deviation (Z = a random process) that is characterized by a zero mean and unit variance (i.e., a random draw from a standard log-normal distribution). The mean sets the general direction for the evolution of the vector of wealth over time, but the evolution of wealth is uncertain because the investment’s volatility generates a range of possibly negative and positive values for each vector component (period return).

The ratio of stocks to bonds will determine the future wealth of the investor. Thus, if we define wealth by the letter ‘w,’ the instantaneous rate of change in the investor’s wealth equals:

$$\frac{dw(t)}{w} = (\mu_S - \mu_B + \sigma_B^2 - \rho\sigma_B\sigma_S)dt + \sigma_S dZ_S(t) - \sigma_B dZ_B(t)$$

where:

ρ = the correlation between stocks and bonds, and,

t = the applicable interval of time.

As stated, any difference between the target portfolio asset allocation (presumed to be optimal for the needs, goals and circumstances of the investor) and the actual portfolio will generate a loss of utility (L). Loss is measured by the degree of tracking error (the squared deviations of the actual portfolio (w(t)) from the optimal portfolio (w*), with the utility cost-per-unit-of-tracking-error measured by the parameter lamda (λ):

$$\text{Utility Loss} = L = \lambda(w(t)-w^*)^2 dt.$$

Over the applicable planning horizon, the investor will want to minimize the discounted (present valued) integral (sum) of the cost of divergence (λ) plus any trading costs associated with the rebalancing function.

³⁷ Brownian motion refers to the movement of molecules in a liquid medium. Movement has a “drift” component and a “diffusion” component. The drift is a central tendency to move in a general direction, the diffusion is the extent to which shocks (i.e., collisions with other molecules) can change the direction of motion. Stock markets, for example, have a drift component that tends to reflect the underlying growth rate of the economy and a diffusion component that reflects exogenous shocks or unexpected surprises.

Leland's model is "self-financing" in that any costs associated with rebalance trades are paid from without rather than from within the portfolio. This makes the mathematics more tractable because any change in the value of the stock allocation (δS) is offset exactly by a change in the value of the bond position with the change having the opposite sign ($\delta S = -\delta B$).

A brief review may be helpful. Leland posits a "no trade" region around the stock and bond positions because the costs of rebalancing small variations from the optimal portfolio may be greater than the cost in "disutility" by not maintaining the strict asset allocation targets of the optimal portfolio. Costs, therefore, involve two terms: a monetary term (κ) measured by trading costs and a utility term (λ) measured by the increased uncertainties of not adhering to the asset allocation target. The total cost function, therefore, includes $\kappa + \lambda$ with κ equal to \$0.00 whenever the asset drift remains within the bounds of the no trade region (W_{max}, W_{min}). Putting it all together, the investor wishes to achieve the most favorable total cost function (V) over the integral of all periods within the applicable planning horizon:

$$V(w(t); W_{max}, W_{min}) = E\left\{\left[\int_{\tau=t}^{\infty} e^{-r(\tau-t)} \lambda(w(\tau) - w^*)^2 d\tau + PV\{\text{transactions costs}\}\right] \text{given } w(t)\right\}$$

This equation says that investor utility is maximized when the present value costs of tracking error plus the present value costs of rebalance transactions are minimized for any given level of wealth.³⁸

Recalling the equation for the instantaneous change of wealth according to the fundamental law of the evolution of wealth listed earlier, Leland "decomposes" it into two terms:

$$a = \mu_S - \mu_B + \sigma_B^2 - \rho\sigma_B\sigma_S; \text{ and,}$$

$$b = \sigma_S^2 + \sigma_B^2 - 2\rho\sigma_S\sigma_B$$

Maximizing the first two derivatives of the total value function leads to the differential equation:

$$awV_1[w; W_{min}, W_{max}] + .5bw^2V_{11}[w; W_{min}, W_{max}] + \lambda(w - w^*)^2 - rV[w; W_{min}, W_{max}] = 0$$

where

r = the risk free rate of return.

Solving the equation produces the optimal parameters for the no trade zone. Leland solves the differential equation for a 60% stock and 40% bond portfolio allocation where:

$$\mu_S - \mu_B = 3.6\% \text{ (expected risk premium of stocks over bonds)}$$

$$r = 7.5\% \text{ (the risk free rate)}$$

³⁸ Leland's argument also assumes that a passive portfolio management strategy (i.e. drifting mix) has a quantifiable cost that can be measured relative to the ideal asset allocation. Thus, rebalancing mitigates "tracking error." This has proved to be a fruitful concept in much recent research. For example, one study notes: "What drives the benefit of rebalancing is reducing the tracking error from getting far off-target. As it happens, tracking error is quadratic. It's proportional to the square of the deviation from the target allocation. For example, when a portfolio with a 30% target for U.S. bonds gets to 32% bonds (2% over target) the tracking error is four times as high as being 1% over target. And if bonds climb to 33% of the portfolio, the tracking error risk is nine times the risk associated with a 1% deviation." Masters, Seth J., "Rules for Rebalancing," *Financial Planning* (December, 2002), pp. 89-93. A further extension of Leland's work is found in Sun, Walter, Fan, Ayres, Chen, Li-Wei, Schouwenaars, Tom, & Albota, Marius A., "Optimal Rebalancing Strategy Using Dynamic Programming for Institutional Portfolios," MIT Working Paper_ssg.mit.edu/group/waltsun/docs/rebalancingSSRN05.pdf

$\sigma_S = 20\%$ (the standard deviation of stock returns)

$\sigma_B = 10\%$ (the standard deviation of bond returns)

$\rho = 0.3$ (the correlation coefficient between stocks and bonds)

$w^* = 1.5$ (the target ratio of stocks to bonds is 60/40)

$\lambda = 0.35$ (derived from maximizing a quadratic utility function)

$\kappa_S = 1.0\%$ (one way transaction cost of trading stocks)

$\kappa_B = 0.5\%$ (one way transaction cost of trading bonds)

Given the above-listed assumptions, the optimal no trade region for the stock position within the portfolio is:

$w_{\min} = 58.69\%$

$w_{\max} = 61.14\%$.

Therefore, if the stock does not break through the upper and lower bounds of the no-trade region, the investor should take no rebalance actions.

Leland's article continues by calculating the optimal rebalance strategy. According to the model, it is optimal to return the portfolio's stock weighting to the edge of the no trade boundary rather than to the 60% allocation target; and Leland compares the costs and estimated turnover of this strategy with a quarterly calendar-based rebalance formula that maintains the exact allocation target. He estimates a cost savings of approximately 50% when the investor uses the optimal strategy.³⁹ Needless to say, if there are other costs (i.e., taxes are considered for taxable investment accounts), the width of the no-trade region expands: "In many cases, the no-trade interval changes with the cube root of the parametric changes."⁴⁰

Many recent mathematically-oriented studies extend model building to encompass a greater range of complex issues—tax costs, a wider variety of risk aversion/utility of wealth functions, the introduction of non-liquid assets into the portfolio (e.g., hedge funds, illiquid annuities, and so forth), and non (log)normal return distributions.⁴¹ A good example of trends in modern research is a study published in 2000 by Lynch (N.Y.U.) and Balduzzi (Boston College).⁴² This study finds that realistic transaction costs:

³⁹ Although trading frequency may increase, Leland's model assumes that trading costs are strictly proportionate and not fixed. Additionally, the amounts traded will be very small. Small trades generate only small costs (unlike the fixed or step-rate trade commission schedule found at most brokerages) and all "costly trades" within the no-trade zone (i.e., trades that produce more trading costs than investor utility increases) are eliminated. The model's predicted cost savings may not be attainable under most wealth administration platforms in today's marketplace.

⁴⁰ Other studies indicate that the nature of the costs (i.e., fixed, proportional, or a combination thereof), determine whether the rebalance action should bring the asset weighting to the closest boundary of the no-trade region or to a point inside the region.

⁴¹ The nature of the return distribution is an important determinate of the suitability of a portfolio management strategy. Normal distributions or random walks are good candidates for rebalancing because the rebalance actions do not affect the expected future portfolio returns. Rebalancing actions within mean-reverting distributions should add substantial value to the portfolio as the investor sells high and buys low. Rebalancing into trending markets, however, can drain money from the portfolio.

1. Cause the rebalancing frequency to decline considerably;
2. Do not materially influence the amounts allocated to risky assets (which, presumably, would trigger more frequent rebalancing because of their relatively high variance); but,
3. Lowers the amount allocated to risky assets if, in addition to rebalance costs, the risky assets have low liquidity.⁴³

Additionally, the Lynch/Balduzzi study has important implications for rebalancing under tactical asset allocation regimes. Specifically, when returns are predictable, the no-trade interval widens considerably and the investor is more willing to incur transaction costs when the boundaries are penetrated. Returns that are random are said to be unconditionally distributed (perhaps approximating a log normal distribution) because the returns are independently and identically distributed (“iid normal”). However, if returns are serially correlated (autocorrelated returns like inflation and certain bond market returns) or mean reverting (like, as some have argued, the risk premium or expected differential between stock and bond market returns), they are said to be predictable.⁴⁴ Economists refer to returns of predictable markets as “conditional” returns.⁴⁵ Although unconditional returns of a market remain unchanged (the standard deviation of historical returns, for example, is not conditional), predictability reduces conditional volatility. Holding unconditional return parameters (expected return, variance and higher moments of the distribution) constant, the Lynch/Balduzzi model explores an investor’s rebalancing behavior given both the magnitude of single-period predictability and the persistence of predictability over several time periods. Lack of persistence, the authors find, causes the no-trade region

⁴² Lynch, Anthony W., & Balduzzi Pierluigi, “Predictability and Transaction Costs: The Impact on Rebalancing Rules and Behavior,” *The Journal of Finance* (October, 2000), pp. 2285 – 2309. See also, Mitchell, John E., & Braun, Stephen, “Rebalancing an Investment Portfolio in the Presence of Convex Transaction Costs,” www.optimization-online.org/DB_FILE/2005/01/1050.pdf. The authors extend the concept of rebalancing “cost” to include a factor for an investor’s confidence in the risk/return forecasts used to optimize the portfolio.

⁴³ Inclusion of non-liquid assets like hedge funds into an investment portfolio cause material difficulties for rational rebalance approaches.

⁴⁴ Even strong advocates of the Efficient Market Hypotheses agree that factors such as dividend yields and the term-structure of interest rates forecast future stock returns, at least in the U.S. markets. See, for example, Fama, Eugene F., & French, Kenneth R., “Business conditions and expected returns on stocks and bonds,” *Journal of Financial Economics* (Vol. 25, 1989), pp.23-49. The question is whether the informational content of the predictor variables (e.g., spread between default free and risky bonds) is quickly incorporated into the pricing of assets or whether there is a lag in price adjustment sufficient to allow an astute investor to exploit the information profitably. Predictability can refer more to the statistical characteristics of the return distributions rather than a forecast of a price level (e.g., the Dow Jones will hit 20,000 prior to the end of the year).

⁴⁵ The return expectation for time $t+1$ is conditioned on the information set available to the investor at time $t+0$, if the information set contains useful predictor variables. Return expectations for time $t+1$ that equal the returns of time $t+0$ (i.e., “no change” expectations) are said to be random walk or ‘martingale’ processes. The U.S. stock market is often characterized as a ‘sub-martingale’ return process with a drift (underlying growth rate of the economy) component and a diffusion (new and unpredictable events) component. Martingale processes are examples of unconditional returns. This information is of interest to investors deciding whether to employ active or passive *investment* management. It is ironic that financial institutions risking their own capital by selling derivative instruments into the marketplace derive the pricing of these instruments through the use of martingale mathematics [the trend of asset prices is not important if the derivative positions can be hedged irrespective of how price moves]. Many of these same institutions, however, when risking their client’s capital, advertise they possess superior stock forecasting and market timing abilities (conditional expectations). Based on these claims, they put wealth management client funds into unhedged (and, sometimes, poorly diversified) portfolios. We suspect that, someday, this may form the basis of interesting litigation.

to change from a band or straight-line boundary to a U shaped region.⁴⁶ Furthermore, investors are more comfortable holding risky assets at wider trade ranges when faced with conditional distributions.

Taxes

Tax issues further complicate the rebalance decision.⁴⁷ One area of complexity is the differential tax rates or differing tax treatment of ordinary income (dividends vs. interest), short-term gain or loss, and long-term gain or loss.⁴⁸ Rebalance actions that harvest losses as opposed to those that recognize gains may be more readily taken.⁴⁹ Gain recognition destroys the time value of tax deferral which is a form of opportunity cost for the investor. Given today's low tax rates, however, the value of the tax deferral may not be great enough to justify portfolio concentration risk⁵⁰ through retention of low-basis assets. One measure of the value of the tax deferral is the amount of tax-payment on the recognized gain discounted by the tax-free investment rate for the applicable planning horizon. For example, an asset sale that triggers a \$1 million long-term gain generates (at 20% cap gains rate) a tax liability of \$200,000. But the gain is an embedded gain that must be paid at some future date (other than a sale after the date of the owner's death). The value of the tax liability deferral for a five-year period is, therefore, the present value of a future payment of \$200,000 discounted by the five-year muni-bond rate. Assuming a tax-free rate of 3%, the value of the deferred payments equals \$172,518. Thus the tax cost of the transaction equals (\$200,000 - \$172,518) \$27,482. When a \$27 thousand tax cost on sale of a \$1 million position (assuming a zero basis) is weighed against the risk of maintaining an underdiversified or misallocated portfolio, the costs of not rebalancing may well outweigh the acceleration of the tax liability.⁵¹

Prudence, Utility Theory, and Portfolio Rebalancing

Under what circumstances might it be prudent to drift? One possible answer to this question is based in utility theory. Most investors prefer more wealth to less wealth (the "non-satiation principle") but have a decreasing marginal rate of satisfaction. Earning an additional dollar produces slightly less satisfaction than losing a dollar produces dissatisfaction. An economist might state that most investors are sensitive to changes in their dollar wealth (they exhibit "risk aversion") while a mathematician might state that investors exhibit increasing utility of wealth curves that have positive first derivatives and negative second derivatives (head upwards but at a constantly decreasing rate). Both sets of jargon

⁴⁶ The shape of the no-trade region (or free boundary problem) is further discussed in Dybvig, Philip H., "Mean-Variance Portfolio Rebalancing with Transaction Costs," (January 2, 2005) <http://www.phildybvig.com/papers/tcost1per2.pdf>; and, Donahue, C., & Yip, K., "Optimal portfolio rebalancing with transaction costs," *Journal of Portfolio Management* (Summer, 2003), pp. 49-63.

⁴⁷ The following discussion draws upon Horvitz, *Supra*, pp. 49-53. For a study of the interrelationships between asset allocation, consumption and rebalancing strategies see, Hughen, J. Christopher, Laatsch, Francis E., & Klein, Daniel P., "Withdrawal Patterns and Rebalancing Costs for Taxable Portfolios," *Financial Services Review* (Vol. 11, 2002), pp. 341-366.

⁴⁸ Not to mention, the propensity of the U.S. Congress to change tax law with great frequency.

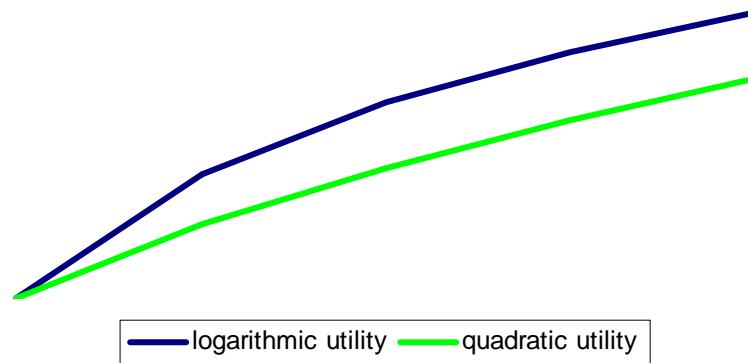
⁴⁹ Berkin, Andrew L., & Ye J., "Tax Management, Loss Harvesting, and FIFO Accounting," *Financial Analysts Journal* (July/August, 2003), pp. 91-102.

⁵⁰ In the jargon of economists, "bloated fractional representation."

⁵¹ This simplified example ignores the uncertainty of future returns on the value of the underlying portfolio assuming no transaction. For a more complete discussion, see Stein, David M., "Diversification of Highly Concentrated Portfolios in the Presence of Taxes," *Investment Counseling for Private Clients II* (AIMR Conference Proceedings, 2000), pp. 18-25; Stein, David M., & Narasimhan P., "Of Passive and Active Equity Portfolios in the Presence of Taxes," *The Journal of Private Portfolio Management* (Fall, 1999), pp. 55-63; and, Abeysekera S.P., & Rosenbloom, E.S., "Optimal Rebalancing for Taxable Portfolios," *Journal of Wealth* (Winter, 2002), pp. 42-49. The interaction between portfolio rebalancing and the value of tax timing options is discussed in Al Zaman, Ashraf, "Asset Location and Allocation with Multiple Risky Assets, (Purdue University, March 16, 2004) <http://economics.ca/2004/papers/0334.pdf>.

mean the same thing: the joy of a \$1.00 gain is not as great as the pain of a \$1.00 loss. There are a variety of different types of curves that can fit into the generalized mathematical model known as von Neumann-Morgenstern utility functions (named after two Princeton University mathematicians).⁵² The following chart illustrates two curves that fit nicely into the von Neumann-Morgenstern family: a

Investor Utility Curves



logarithmic curve (Utility = log of wealth) and a quadratic curve (Utility = square root of wealth):

The investor with the logarithmic utility curve is said to be more sensitive to changes in wealth because the curvature produced by the function is greater.⁵³ Chances are that the portfolio that will best satisfy the log of wealth investor will not be the portfolio that will best satisfy the quadratic utility investor.

⁵² Von Neuman and Morgenstern explored the mathematics of choice in uncertain situations, and developed a series of axioms that characterize rational choices. For example, if the utility of x is preferred to y, and the utility of y is preferred to z, then the utility of x must be preferred to z. In an investment framework, the expected utility of wealth $E[U(W)]$ is denoted as follows:

$$E[U(W)] = \sum p_i U(w_i)$$

where 'p' represents probability of attaining wealth and 'w' represents the amount of wealth in state 'i.' The sum of the probabilities are equal to one, thus the probability-adjusted utility of wealth equals the utility value of wealth generated under a variety of economic conditions (a dollar in a recessionary economy may have a greater utility than a dollar in an prosperous economy) adjusted for the probability of the occurrence of a specific economic state (recession, stagflation, growth, deflation, etc.). Harry Markowitz (winner of the Nobel Prize in economics for development of the fundamentals of Modern Portfolio Theory) drew upon Von Neuman and Morgenstern's work but assumed that investment distributions were normal or bell-shaped distributions fully determined by their risk (standard deviation) and expected return. Investors are, therefore, assumed to have quadratic utility—that is to say, only two elements (mean and variance) affect their investment choices. Mean is a first order term and variance is a squared term; hence, utility (U) can be represented by a quadratic equation $U = a\mu + b\sigma^2$. Using the Von Neuman and Morgenstern axioms, Markowitz derived the following utility functions: If the utility of a certain result (say a guaranteed return on a Certificate of Deposit) is greater than the expected utility of wealth over a variety of possible but uncertain outcomes (the expected return of which equals the certain return) $\{U[E(W)] > E[U(W)]\}$, then the investor is risk averse;

If the utility of expected wealth equals the expected utility of wealth $\{U[E(W)] = E[U(W)]\}$, then the investor is risk neutral; and,

If the utility of expected wealth is less than the expected utility of wealth $\{U[E(W)] < E[U(W)]\}$, then the investor is risk seeking.

Markowitz assumes that rational investors are risk averse. In the Markowitz world, the investor with quadratic utility prefers a certain \$400 (utility = square root of 400 = 20) to a risky proposition offering, at 50/50 odds, either \$100 (utility = square root of 100 = 10÷2 = 5) or \$700 (utility = square root of 700 = 26.46÷2 = 13.23). Despite the fact that the uncertain proposition has the same expected value as the sure thing, the certain result is preferred because of its higher utility: 20 is greater than 5 + 13.23, or 18.23.

⁵³ It is the curvature of the lines rather than their relative placement on the graph that is of importance.

The reason for this preliminary conclusion lies in the fact that these investors have very different views about risk and reward. Some investors have utility curves that are straight line (linear with respect to wealth). These investors are 'risk-neutral.' Others exhibit "gamblers' curves" that monotonically decrease instead of increase. For gamblers, the thrill of the wager often has greater utility than the level of expected wealth offered by the wager.

The mathematics of utility and risk aversion curves is interesting and important.⁵⁴ Quadratic utility assumes that the investor becomes more risk averse as wealth increases. This means that the investor exhibits Absolute Risk Aversion [ARA] because a 5% negative return causes a millionaire to lose more dollars than a 5% negative return causes a child with a \$100 Christmas club account to lose. Although this type of utility curve may seem counterintuitive, nevertheless, it is characteristic of a certain group of investors.⁵⁵ Many investors have utility curves that exhibit decreasing absolute risk aversion and Constant Relative Risk Aversion [CRRA].⁵⁶ Finally, there are investors with "kinked" risk aversion curves. They may feel comfortable with investment risk above a specified portfolio value, but may become highly risk averse below the threshold value.⁵⁷ The important point to note is that some investors are highly affected by shifts in wealth while others may remain largely unaffected. Returns measured in dollar space are no longer adequate gauges of portfolio performance. Rather, performance is best measured in utility space.

For investors who exhibit a high degree of sensitivity to shifts in wealth (increased risk tolerance when wealth increases and decreased tolerance when wealth declines) a passive or drifting portfolio management strategy may be appropriate. For a two-asset class portfolio (stocks and bonds), when stocks increase in value (i.e. assume a greater proportion of weight in the portfolio), the investor can tolerate the increased risk. However, when stocks decline in value (i.e., the portfolio loses wealth), they constitute a lower percentage of portfolio weight relative to the safer bonds. But this is exactly what the investor may wish. Along the spectrum of possible risk aversion curves, highly risk-averse investors may prefer an insured portfolio strategy; while, at the other end of the spectrum, an investor with low risk aversion may wish to employ a strict contrarian strategy. Between these two extremes lie a series of portfolio management elections that include (1) the tactical asset allocation strategy⁵⁸ (appropriate for investors that do not exhibit quite the extreme reaction to wealth changes as the highly risk-averse investor who may prefer portfolio insurance); and, (2) the elections to rebalance towards the strategic asset allocation targets. Rebalance elections are appropriate for investors that exhibit risk aversion curves more in line with the "average" within the population of investors.⁵⁹ However, for some investors, it may be prudent to drift!⁶⁰

⁵⁴ This topic is beyond the scope of the essay.

⁵⁵ One thinks of a business person who takes much risk building the enterprise. At the time of sale, however, the entrepreneur may exhibit great aversion to taking investment risk.

⁵⁶ Such curves are consistent with investors that exhibit log utility of wealth. Constant risk aversion means that both the millionaire and the child with the Christmas club account would be willing to risk 5% of their wealth given a reasonable expectation of investment gain. Most mathematical models of portfolio choice assume that investors have CRRA curves.

⁵⁷ This behavior is consistent with defined benefit pension plans that seek to maintain a plan surplus. A comparable set of behavior may be exhibited by an individual who is risk averse until a wealth target is reached; but, when the goal is attained, the investor becomes more risk seeking with the excess money.

⁵⁸ For a review of the historical results of tactical asset allocation vs. fixed mix strategies see, Arshanapalli, B., Coggin, T.D., & Nelson, William, "Is Fixed-Weight Asset Allocation Really Better?" *The Journal of Portfolio Management* (Spring, 2004), pp. 27-38. The authors compare the asset allocation recommendations of eight major brokerage firms and conclude "when we apply a strict statistical test, none of the eight brokers is significantly different from the [fixed weight allocation]."

⁵⁹ For a more complete discussion, see Arnott & Lovell "Rebalancing: Why? When? How Often" *Supra*, pp. 9-10. See also, Masters, Seth J., *Supra*: "If an investor is extremely risk-tolerant or, more precisely willing to endure higher risk for higher potential returns, then the benefit of rebalancing is smaller than it would be for an investor who is less risk tolerant."

A Simulation Model

In order to gain insight into the economic consequences of rebalance elections for taxable investors owning diversified portfolios administered with realistic costs, we simulate asset management results over three regimes:

1. Periodic distribution on a percentage-of-corpus (unitrust) basis over a 360 month planning horizon;
2. Periodic distribution on an indexed annuity basis over a 360 month planning horizon; and,
3. Accumulation basis with no cash flows over a 360 month planning horizon.

The two distributional regimes may represent retirement income elections, spending rules for endowments, trustee distributions from private trusts, and so forth. In contrast to asset 'decumulation,' the accumulation regime may represent the investment of an inheritance, lottery winning, large gift, or any one-time, long-term commitment of capital.

The model tests nine rebalance strategies across each of the three regimes. Specifically, it considers:

1. Drifting mix (no rebalancing);
2. Rebalancing yearly to the target asset allocation;
3. Rebalancing yearly halfway to the target allocation;
4. Rebalancing to target allocation when an asset position is off by 5%;
5. Rebalancing halfway to the target allocation when an asset position is off by 5%;
6. Rebalancing to target allocation when an asset position is off by 10%;
7. Rebalancing halfway to the target allocation when an asset position is off by 10%;
8. Rebalancing to target allocation when an asset position is off by 15%; and,
9. Rebalancing halfway to the target allocation when an asset position is off by 15%.

When the rebalance trigger is a percentage change in asset value relative to portfolio value (i.e., rules 5 through 9), all investment positions are rebalanced according to the specified formula.

The portfolio is owned by a taxable investor who incurs a 35% ordinary income tax liability on interest, dividends, and short term gains, and a 15% capital gains tax liability for realized long term price appreciation. The tax basis of each investment within the portfolio is assumed to be 50% of the initial fair market value of each position. The portfolio's aggregate initial value equals \$1 million. The model's output shows both pre and post portfolio liquidation results. The pre-liquidation results are proxy outcomes for investors who, at death, pass portfolios with a step-up in basis to heirs, for trusts making in-kind terminating distributions to remaindermen, for certain intervivos and post-mortem gifts to charities,

Masters develops a rebalance formula that incorporates a term for costs and a term for investor risk aversion. He concludes that the optimal rebalance strategy is "halfway between the trigger point and the initial target allocation."

⁶⁰ A similar conclusion was reached in a study that compared the utility value of rebalance strategies to investors with dissimilar utility of wealth functions. See, Clark, Truman A., "Efficient Portfolio Rebalancing," www.dfafunds.com (Fall, 2001).

and so forth. The post-liquidation results are proxy outcomes for investors making taxable transfers of assets without benefit of a basis adjustment, for trusts liquidating the portfolio to make cash distributions to the remaindermen, or for noncharitable bequests. The tax liabilities faced by the investor may dictate which rebalance elections convey the greatest portfolio management benefits.

The model assumes that the investor exhibits maximum tax aversion. Tax aversion can be expressed in one of two ways:

1. A desire to avoid realizing short-term gains in favor of long-term gains; and,
2. A desire to avoid selling low basis asset positions in favor of high basis asset positions.

There is no practical way to attach a holding time to a specific investment position and, therefore, it is impossible to implement simulated dynamic short-term versus long-term preferences. Time-based sale preference is fixed statically at the beginning of the simulation by specifying which percentage of gains will face short term versus long-term tax rates. The static percentage is then applied to all asset sales. Basis, however, can be simulated. Basis preferencing is modeled through the use of the variable "Aversion to Realizing Gains," which ranges from 0 (no aversion) to 100 (maximum aversion). In its simplest form, aversion operates as the percentage of realized sales with zero capital gain versus the percentage of realized sales whose basis is equal to the average basis for the asset class. However, it is clear that as the proportion sold increases, this formula breaks down. In the extreme case, the entire sale of an investment must take place at the average basis for that investment. The derived basis for a given sale is modeled as the simultaneous solution to a set of linear constraints determined by:

1. The tax aversion percentage;
2. The required net cash flow for a given period;
3. The current basis of the asset class; and,
4. The blended tax rate (ordinary income + capital gains) applied to asset sales.

The realized gain will approach zero as the tax aversion increases and the percentage of assets sold decreases, while realized basis will approach average basis as the tax aversion decreases and/or the percentage of assets sold increases. Tax aversion is not, in the interest of maintaining a diversified portfolio, used to prefer the sale of one asset over another. Such preferences would soon result in the elimination of high basis asset classes from the overall portfolio, as they would be overwhelmingly favored for sales.

The test portfolio has a 70% weighting to stocks and a 30% weighting to bonds. Investment positions are proxied by indexes such as the S&P 500 stock index, CRSP small stock indexes, EAFE index of foreign stock, Lehman bond indexes, and so forth. The following table provides details of the asset allocation targets as well as (1) the ordinary income component of simulated return; and, (2) the assumed yearly turnover absent rebalancing transactions or sales to fund distributional requirements.

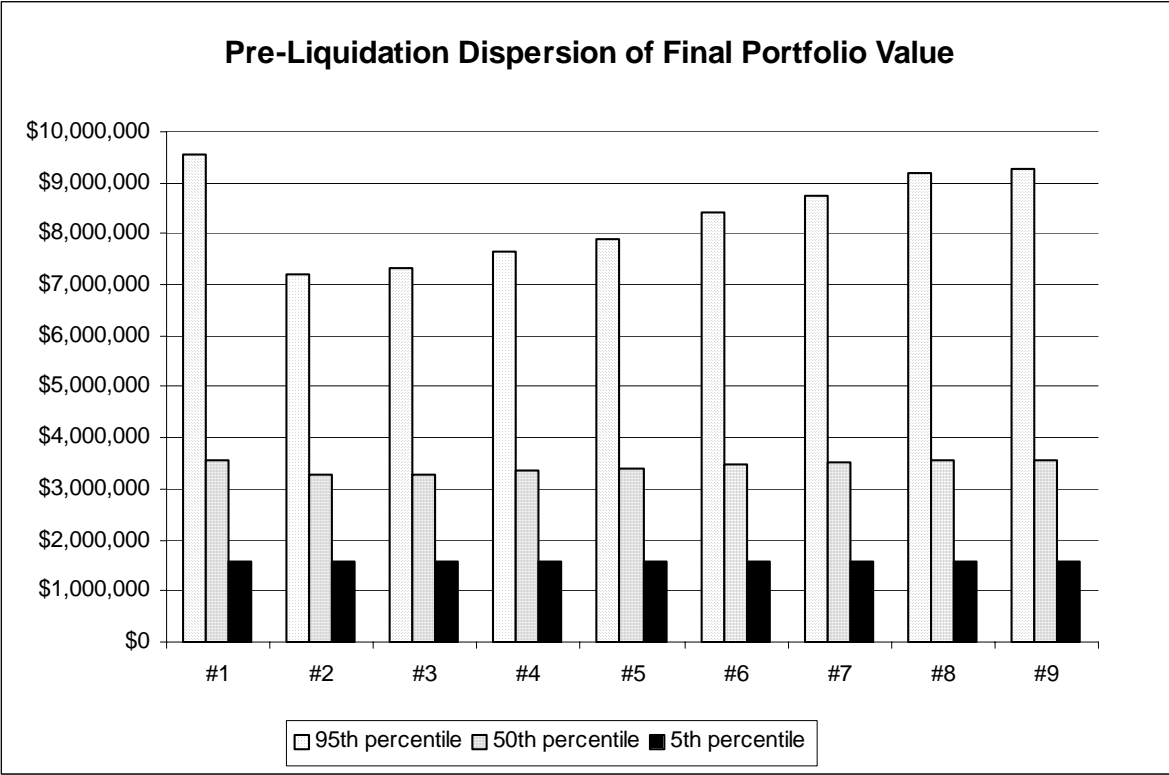
Equity	Weighting	Income Yield	Annual Turnover
US large company stocks	\$300,000	2.0%	5%
US small company stocks	\$100,000	1.5%	20%
US micro cap stocks	\$50,000	1.5%	20%
International large company stocks	\$150,000	2.5%	15%
International small company stocks	\$50,000	2.0%	15%
Emerging Market stocks	\$25,000	2.0%	10%
US Securitized Real Estate	\$25,000	4.5%	10%
Fixed Income			
US Short-term fixed income	\$100,000	3.0%	100%
US Intermediate-term fixed income	\$100,000	5.0%	100%
Global Bonds	\$100,000	3.0%	100%

The portfolio pays all costs, charges, and investment expenses. The model assumes that the investor incurs a \$5,000 minimum annual fee. In any year the portfolio bears the greater of the minimum annual fee or a fee schedule equal to 1.10% on the first \$1 million in assets and 0.90% on the excess. These costs may reflect investment advisor fees, mutual fund expense ratios, wrap account costs, or other types of charges characteristic of wealth management platforms found in today's marketplace. It is important to note that expenses are charged to the portfolio on top of the amounts disbursed under the portfolio's distribution policy. This means, for example, that a portfolio distribution policy calling for a disbursement of \$50,000 per year indexed for inflation (an indexed annuity regime) requires that the portfolio liquidate assets sufficient to fund the investment costs, the tax liabilities, the \$50,000 annuity and, finally, the amount of gross-up required to maintain the annuity's purchasing power. Annual distribution targets are disbursed monthly (and, taxes and expenses are calculated and deducted monthly) and results are expressed in constant dollar terms (i.e., adjusted for inflation). Constant dollar reporting is a present-value calculation with the year-by-year discount rate equal to inflation.

The portfolio evolves over 30 years according to 5,000 simulated economic conditions ("trials"). The predictor or 'state-of-nature' variable for future economic conditions is the month-by-month change in the value of the Consumer Price Index (CPI). CPI is modeled as an autocorrelated variable that, in turn, correlates to the time series of asset class returns. Although return data on each investment class was not available from 1973 through 2004, this period was used to calculate the variance/covariance matrix used to model the evolution of the portfolio. The results of the 5,000 trials are arranged from highest to lowest values. The "best case" trail is the value of the 4,750th (95th percentile) highest-value trail; the "median case" is the value of the 2,500th (50th percentile) highest-value trials; and the "worst case" is the value of the 250th (5th percentile) highest value trial. Thus, results are reported within a 90% confidence interval with a 5% chance of achieving a result more favorable than reported and a 5% chance of achieving a result less favorable than reported.

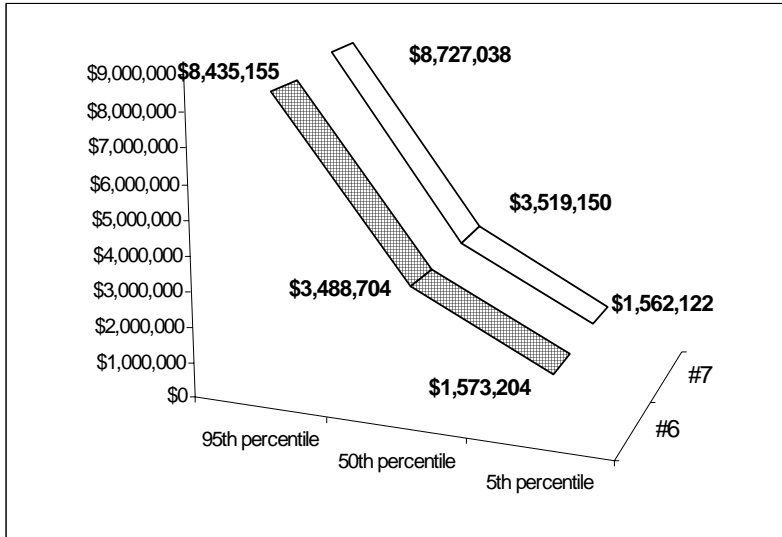
Rebalancing Strategies During the Accumulation Phase

Turning first to the accumulation period (i.e., no cash flows), the following chart compares the pre-liquidation range of terminal values produced under each rebalance strategy.



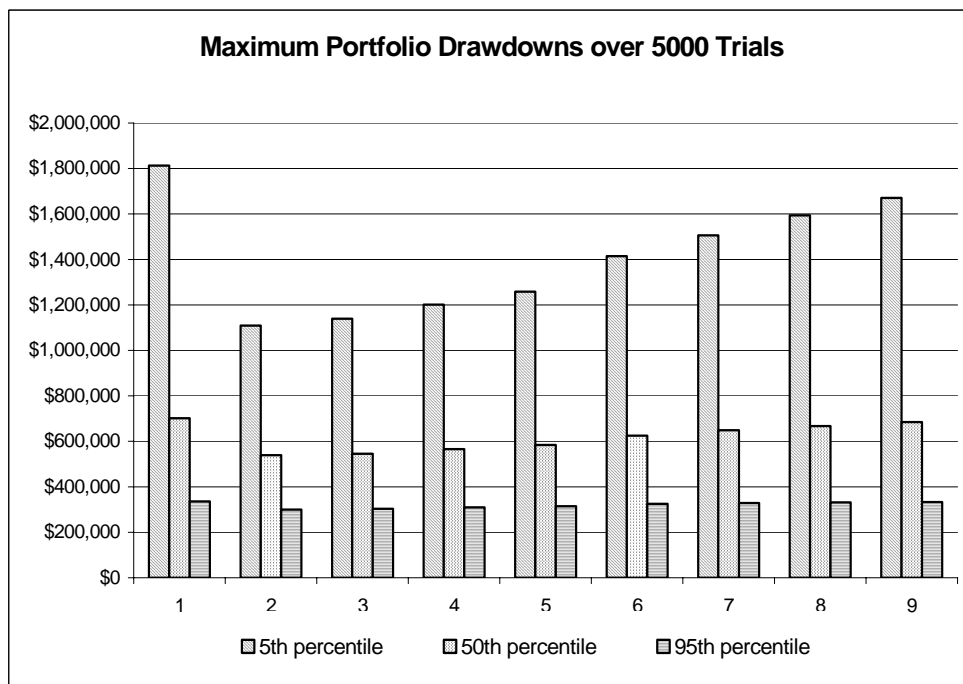
The chart’s single-dimensional view of final constant dollar results provides mixed evidence regarding the effects of rebalancing in a taxable environment. No portfolio produces first-order dominance (higher values at the 5th, 50th and 95th percentiles of the distribution). Strategy number 1 (drifting mix) produces the greatest dispersion in terminal wealth with both the highest median (50th percentile) value (\$3,572,552) and the highest upside boundary value (\$1,000,000 evolves to \$9,554,251 in 30 years).

Strategy #4 (rebalancing to the target at a 5% drift threshold) produces the most favorable downside result (\$1,575,943). However, the range of downside results across all rebalance elections (\$1,575,943 - \$1,560,127) is an economically insignificant \$15,816. Consideration of results at both the median and upside boundary indicates that widening the width of the no-trade region from a 5% to a 15% threshold produces results that approach those of the drifting mix portfolio. Considering terminal wealth results only, one has to look closely to evaluate several of the predictions produced by theoretical models or allegations suggested by historical empiricism. Consider, for example, the results of strategies 6 and 7 under which the investor elects to rebalance the portfolio at a 10% drift threshold (#6 to target allocation / #7 halfway to target allocation).



Rebalancing back to the target allocation “tightens” the distribution of terminal wealth by truncating wealth at the upper bound wealth by \$291,883 and increasing wealth at the lower bound by \$11,082. Therefore, the simulation model suggests that there may be some risk control benefits to more stringent rebalance strategies. The expected utility of the tighter risk control, however, may not be positive for some investors because of the reduction of both median and upper bound wealth.

The risk-control benefit of portfolio rebalancing is more clearly seen under a variety of downside risk measures. The following chart compares the maximum drawdowns experienced under various rebalance strategies. In this case, the 5th percentile represents the highest-value drawdown or, the “worst” results; the 95th percentile represents the lowest-value drawdown or the “best” results.



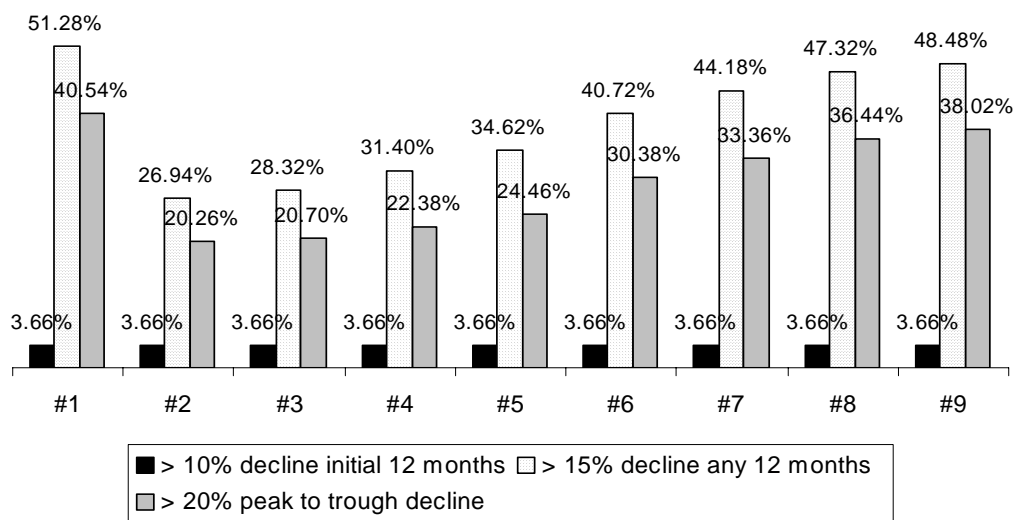
Under this risk metric, the drifting mix portfolio is clearly dominated by all rebalance strategies. Across 5,000 trials, the drifting mix (strategy #1) had the greatest dollar value declines throughout the distribution of results. Not surprisingly, as the threshold for rebalancing is pushed back from 5% to 15%, the magnitude of drops in portfolio value at the benchmark percentiles of the distribution approach those experienced under a purely passive or “benign neglect” portfolio management approach.

The promise of greater terminal wealth after a 30-year accumulation period may not be sufficient to permit the investor to stay the course. As suggested, an investor’s risk tolerance is a function both of the investor’s ability to assume risk and the investor’s willingness to do so. Trusts and Endowments, for example, may have the ability to assume risk but their fiduciaries may not be willing to assume “agency” risk in which their careers may be jeopardized or their personal assets subject to surcharge because of allegations of breach of fiduciary duty.

The model evaluates the probability of the following downside risks over the planning horizon for asset accumulation:

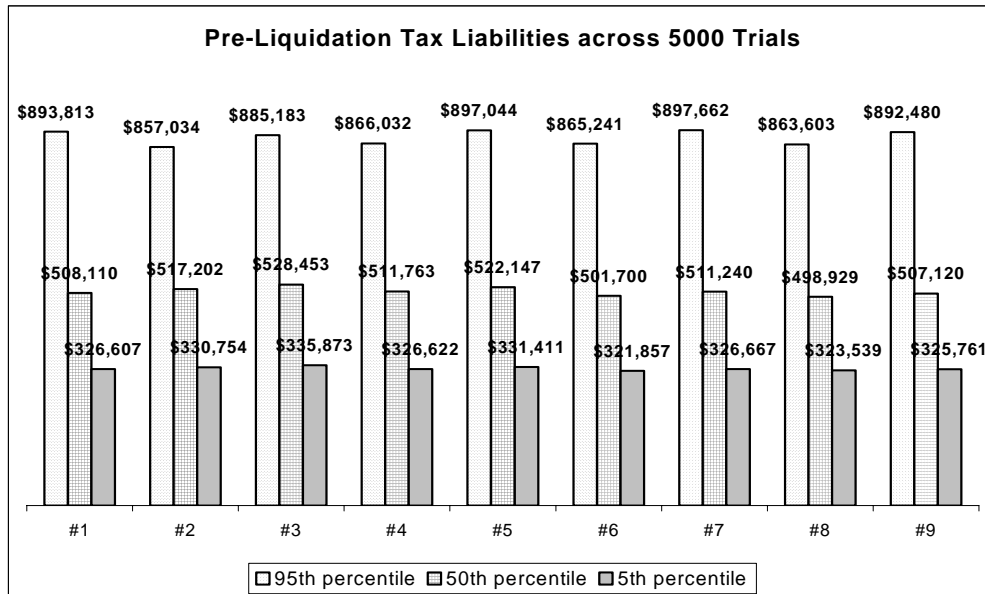
1. Probability of a decline in value of 10% or greater during the initial 12 months of portfolio operation;
2. Probability of a decline in value of 15% or greater during any 12 month period; and,
3. Probability of a decline in value of 20% or greater from a peak to a trough.
4. Results appear on the following graph:

Downside Risk Probabilities

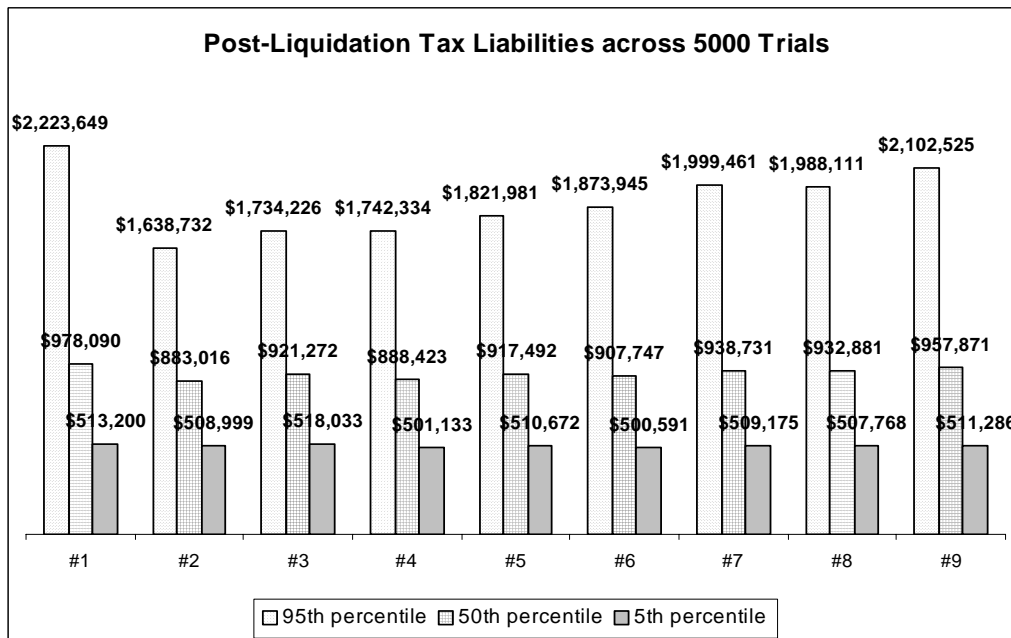


The above chart clearly demonstrates the value of rebalancing as a mechanism to control risk. The probability of adverse occurrences increases systematically as the investor moves from a calendar-based rebalance formula through threshold formulae and, finally, to a strategy of portfolio drift.

Although there appears to be some relationship between taxes paid and rebalance elections, it is not particularly strong. In general, the range of tax liability is similar for each strategy. As a general rule, for any given formula, full rebalancing generates a slightly lower cumulative tax drag on portfolio growth. Although this result may be somewhat counterintuitive, it may reflect the fact that partial rebalancing transactions must occur more frequently when volatile assets are included in the portfolio. The following chart depicts cumulative constant-dollar tax liability for pre-liquidation portfolios:



Tax liability assumes an increased importance, however, when one considers results for post-liquidation portfolios. For example, at the upper bound results, the magnitude of the embedded tax liability generated under each rebalancing election differs considerably. The following chart provides insight into the range of total taxes paid when the portfolio is liquidated at the end of the accumulation period.



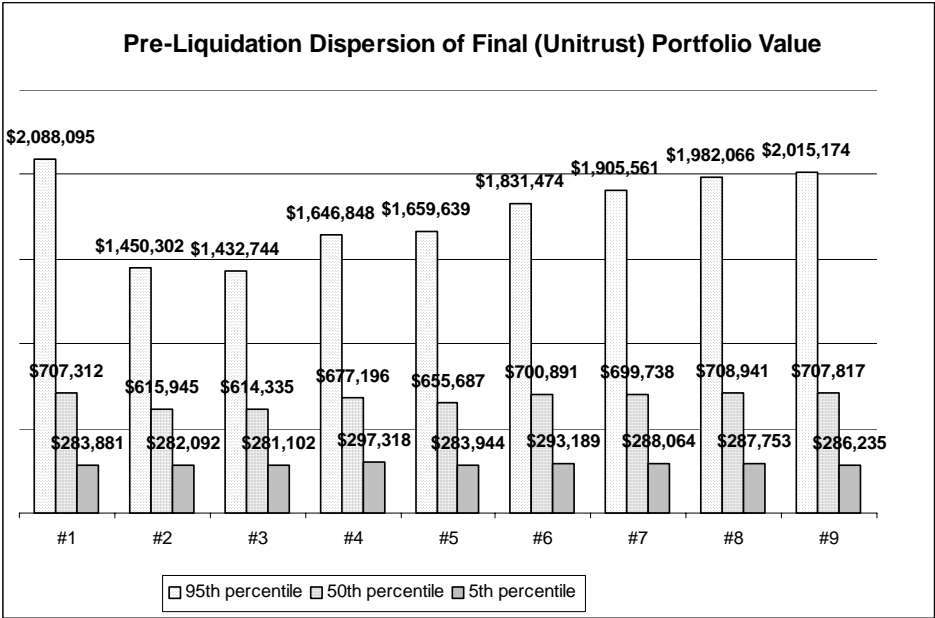
Rebalance Strategies During the Decumulation Phase: Unitrust Distributions

In general, rebalance elections under a regime of asset decumulation evidence a more pronounced grouping of results. That is to say, the necessity to finance required distributions, taxes and fees by periodic liquidation of investment positions mitigates much of the effects of active portfolio management decisions. The investor simply liquidates more of the asset with a high percentage-of-portfolio value and less of the other assets. Rebalancing “tags along” with the portfolio’s cash flow requirements (spending policy, fee and expense payments, and tax costs). This paper presents the model output under two 30-year decumulation regimes:

1. A unitrust distribution policy under which the portfolio disburses 5% of its average value calculated over the previous 36 month period (or the average value over all months for periods prior to the initial 3 years); and,
2. An indexed annuity distribution policy under which the portfolio disburses the inflation-adjusted equivalent of \$50,000 per year.

Whereas the portfolio now operates under both an investment policy and a spending policy, the model also tracks the consequences of rebalance elections on inflation-adjusted aggregate consumption over the planning period. Finally, it considers both pre- and post-liquidation results at the end of the 30-year planning horizon.

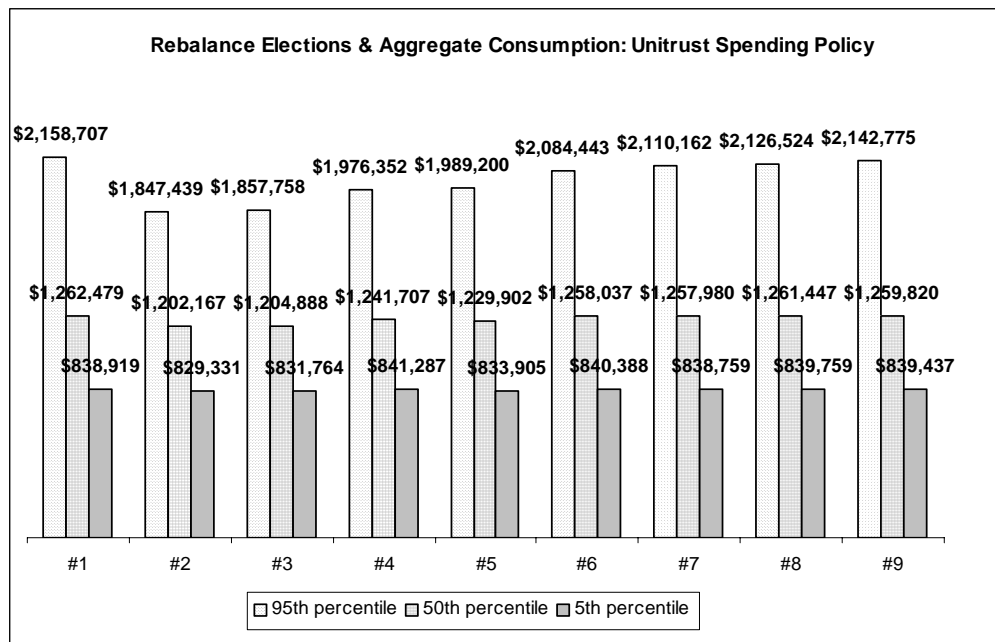
The following charts compare and contrast final pre-liquidation portfolio values with total consumption over a 30-year horizon. They depict results under the unitrust regime. For an individual investor consuming assets throughout retirement, final pre-liquidation portfolio value may be a proxy for the inflation-adjusted value of a bequest where the portfolio receives a step up in basis; for a trustee, final value may represent the value of a remainder interest for a terminating family trust where the trustee distributes assets in kind. Likewise, total consumption may represent consumption achievable during retirement income for an individual investor; or, the interests of a current beneficiary under a percentage-of-corpus trust distribution policy.



Under the unitrust distribution regime, the simulation model’s output fails to reveal first-order dominance with respect to terminal wealth. The ‘no-rebalance’ strategy (#1) produces the most favorable outcome at

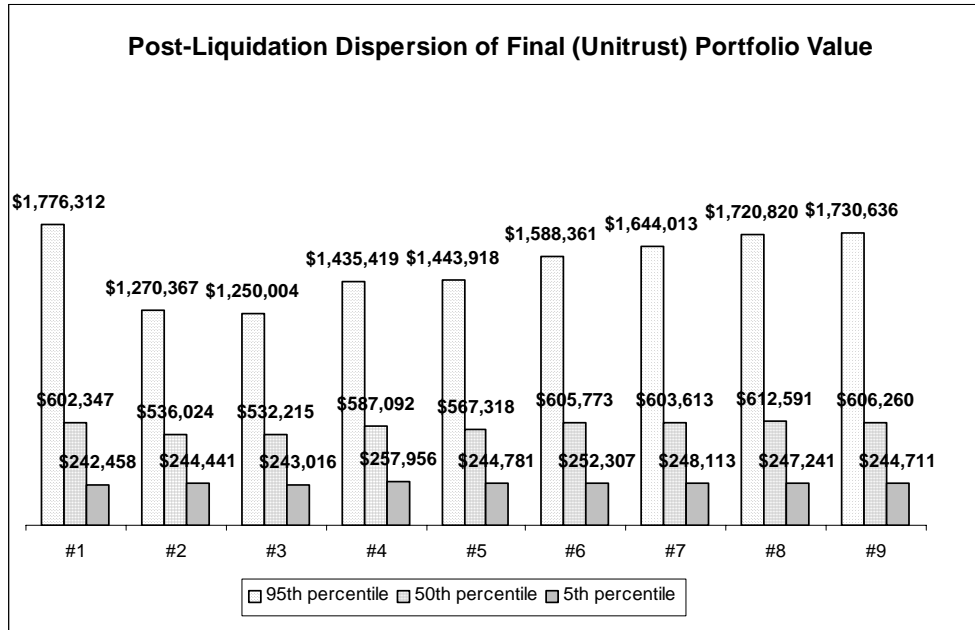
the upper boundary. This, however, should not be surprising because the drifting mix has, over time, the expectation of an increased equity weighting. Higher terminal values may simply reflect greater investment risk of the increased equity commitment. As the threshold for triggering a rebalance action widens, the median (50th percentile) results approach the median result for the drifting mix portfolio management strategy. Indeed, at a 15% drift threshold, the rebalanced portfolios produce, on average, slightly more favorable results than a purely passive approach to portfolio management. Finally, the simulation analysis suggests that the “best case” at the lower bound (5th percentile) is produced by a strategy electing to rebalance to the targeted allocations at a 5% drift threshold (strategy #4). Lower bound differences, however, do not appear to be economically significant.

The following chart depicts aggregate consumption under a unitrust spending policy over the planning period:



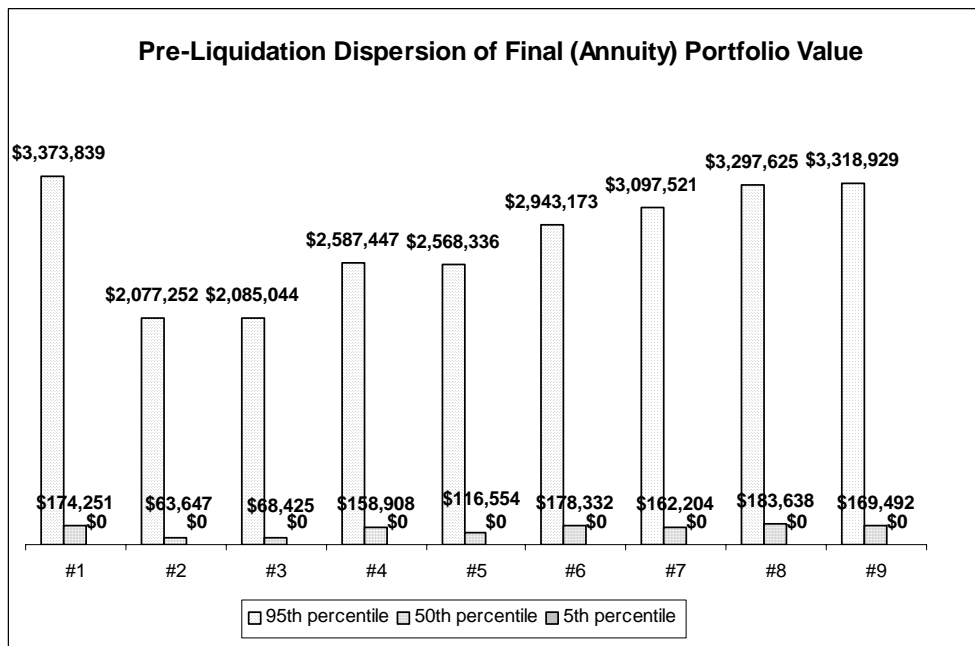
It is interesting to note that even over a 30-year period, a portfolio with an initial value of \$1 million does not necessarily allow its owner to achieve \$1 million consumption under a unitrust election. Only when the terminal value of wealth is added to aggregate consumption, at the lower bounds, does the future receipt of dollars equal the present value of those dollars. Finally, with respect to maximum drawdowns, downside risk measures, and total taxes paid over the period, the simulation model produces tightly grouped results. The paper’s appendix provides a more detailed summary of results.

Finally, it is interesting to compare the pre-liquidation portfolio value of the unitrust spending policy with the post-liquidation value. As expected, the magnitude of the embedded tax liability under each rebalance election in the unitrust distribution regime is not as great as the embedded liability found in the accumulation regime. The constant liquidation of investment positions to fund distributions and other costs triggers an ongoing series of smaller tax liabilities.

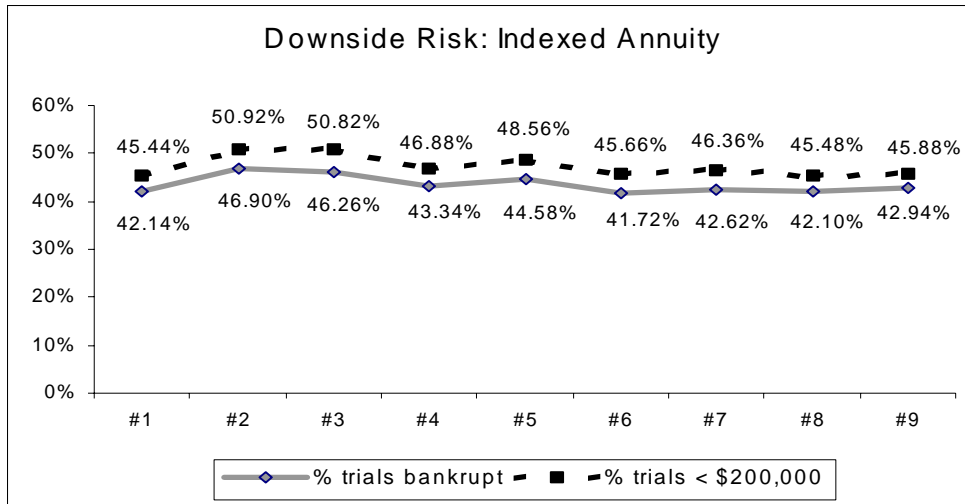


Rebalance Strategies During the Decumulation Phase: Indexed Annuity Distributions

As one turns from the unitrust distribution election to the indexed annuity election there is a considerable change in terminal wealth results. Across all rebalance strategies the lower bound results are portfolio bankruptcy. The combined stress of the indexed annuity withdrawals, taxes and other costs deplete the portfolio prior to 30 years. This event occurs for the drifting mix portfolio management strategy in month 224. However, the other rebalance strategies evidence little or no ability to combat bankruptcy risk (there is only a 4 month difference with respect to the month in which the portfolio becomes bankrupt between the nine strategies). Results are as follows:

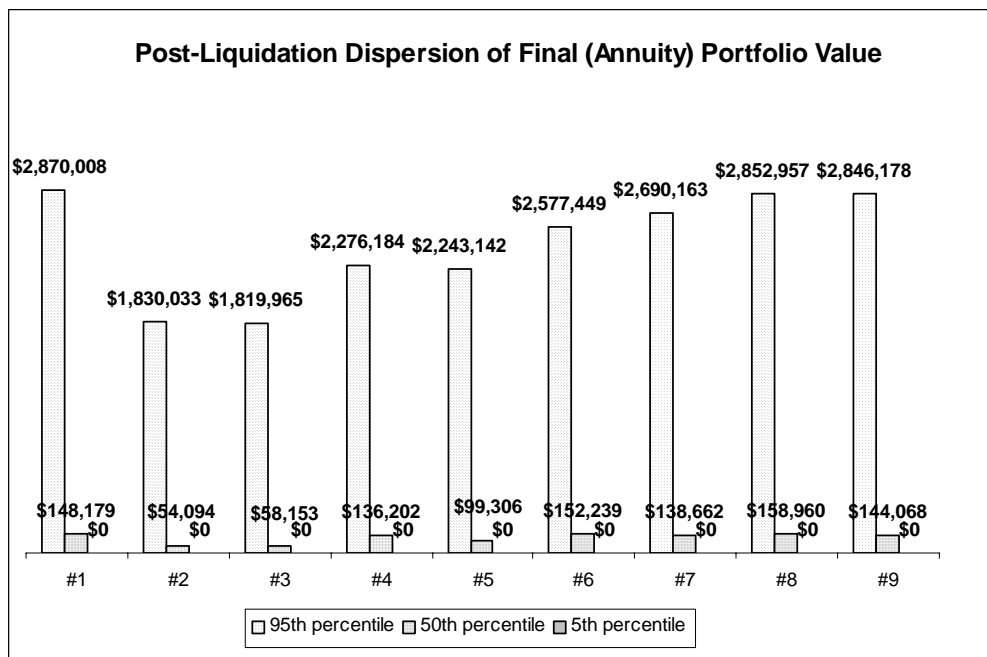


The likelihood of bankruptcy prior to the end of the planning period is high for all portfolio management strategies under the indexed annuity election which sets distributions at \$50,000 per year (or 5% of the initial \$1 million portfolio value). In some cases, rebalancing formulae slightly exacerbate the problem. The following chart shows the likelihood of bankruptcy prior to the end of 360 months as well as the likelihood of penetrating a floor value of \$200,000 (in nominal dollars rather than constant dollars).



Although, on average, the portfolio provides level, inflation-adjusted consumption throughout the applicable horizon, there exists a very high failure probability. This information should be especially valuable for investors entering into a period in which they wish to spend funds from the portfolio. Although the notion of financing a yearly \$50,000 distribution from a \$1 million portfolio may not seem daunting, the above results should give pause to investors/trustees who must sustain purchasing power over long time periods.

Not surprisingly, the following chart illustrates that post-liquidation values under an indexed annuity spending regime can also be anemic.



It is important to keep in mind that:

1. The model's 30-year planning horizon may be longer than that faced by many retirees (e.g., those approaching their eighties or nineties);
2. The asset allocation may not be optimal for the planning horizon and distribution stress placed on the portfolio;
3. The portfolio generates various costs including expenses, fees and taxes which, in addition to distributions, are paid in full from the portfolio; and,
4. Models using index returns unadjusted for portfolio frictions often are wildly over-optimistic in terms of projections of feasible spending targets.

The high bankruptcy rate is not an argument for the unsuitability of an indexed annuity distribution election or an argument that a unitrust distribution election is preferable. Rather, it is a caution that such any spending or asset management election carries with it certain risk/reward tradeoffs that must be carefully weighted. The paper's appendix, for example, indicates that the indexed annuity election, across all rebalance elections, provides, on average, more aggregate consumption over the planning horizon. It is difficult to extract any financial planning rules from the model's output. One could easily argue that an investor with a high consumption / low bequest preference could prudently adopt an indexed annuity spending policy over longer planning horizons provided that other income sources (e.g., reverse annuity mortgage) were available if the portfolio faced depletion. Perhaps the best financial planning rule is that portfolios facing dynamic liabilities should be managed under adaptive investment policy (a 'systems engineering' approach) rather than under static investment policy (an 'architectural' approach).⁶¹

Conclusion

Investors and fiduciaries would like to know the probable consequences of portfolio management elections prior to their implementation. This is especially the case for decisions to incur voluntarily extra costs and taxes by electing to rebalance the portfolio to its asset allocation targets. Fortunately, in today's lower tax-rate environment, the tax drag on portfolio accumulations is relatively less severe. However, payment of \$500,000 or more in taxes (on a present valued basis with inflation acting as the discount rate) on a portfolio with an initial value of \$1 million is a realistic possibility.

The setting of realistic accumulation and consumption goals in a taxable environment is an important aspect of investment planning. Furthermore, the growth of wealth is not free. It occurs on platforms that require various costs including commissions, bid/ask spreads, mutual fund fees, wrap fees, market-impact costs, and so forth. Understanding and controlling these costs is also a vital component of controlling a portfolio's risk/reward tradeoffs. A portfolio may fail not because its initial capital is insufficient to discharge reasonable economic objectives; rather, it may fail because insufficient attention was devoted to anticipating and overcoming various portfolio frictions. Tax avoidance, however, may not be a panacea. The naïve investor who accepts the fact that the S&P 500 produced a double-digit return over the past 50 years may be left to ponder how buckets of money evaporated unnoticed out of a high cost tax-deferred variable annuity investment platform.

The literature discussing rebalancing is extensive. Various authors claim that rebalancing elections can control portfolio risk, enhance returns, or both. However, there are few studies examining the economic consequences of rebalance elections for well-diversified portfolios operating with realistic costs and differential tax rates for ordinary and capital gains income. Even fewer studies consider rebalancing elections under both accumulation and decumulation regimes. This paper, following a review of the

⁶¹ Collins, Patrick J. "A Risk Primer for Investment Fiduciaries: With Special Attention to the Management of Endowment Funds," *California Trusts and Estates Quarterly* (Fall, 2002), pp. 4 - 24.

literature, examined the output of a simulation model designed to test the marginal effects of rebalance elections on terminal wealth, aggregate consumption possibilities, and downside risk measures. The marginal effects are, in some cases, pronounced during the portfolio accumulation period. Moving, for example, from the drifting mix asset management election to an asset management policy calling for yearly rebalancing to a target allocation, reduces the likelihood of a 15% decline in portfolio value during a 12-month period from 51.28% to 26.94%. During the decumulation period, however, the realized risk-reduction benefit is smaller, albeit still substantial: 96.20% v. 84.06% for a unitrust election and 95.78% v. 88.12% for an indexed annuity election. Evidence for enhanced investment returns is mixed. Some rebalance strategies add small amounts of value to average (median) and to lower-bound results. Drifting mix strategies, not surprisingly, produce generally better upper-bound results; but, such results are not a free lunch because they reflect increased risk as the portfolio evolves over longer planning horizons.

The right strategy depends primarily on investor utility. This is especially true for asset decumulation regimes. If unspent terminal wealth (final portfolio value) has value because of bequest preferences or remainder interest considerations, then the investor will select distribution policies and rebalance strategies that will augment the utility of final dollar values. If however, the investor does not have these preferences, the unspent money may merely represent lost consumption opportunities and may produce disutility. In this case, asset management and rebalance strategies may give way to actuarial solutions such as immediate annuities. This is a complex extension of this paper, however, and, given the current transaction-biased and commission-oriented marketplace—which is replete with potential counterparty risk—for actuarial instruments, is itself a solution that may bear considerable risk.

APPENDIX: SUMMARY RESULTS

Summary of Portfolio Rebalance Elections During Accumulation Period \$1 million starting portfolio / 70-30 allocation. End of 360 months. Values Adjusted for CPI.

Preliquidation Values (Stepped Up Basis at Death / Distribution in kind / Bequest to Charity)

	#1 No Rebalance	#2 Rebalance Yearly to target	#3 Rebalance Yearly halfway to target	#4 Rebalance to target at 5% drift	#5 Rebalance halfway to target at 5% drift	#6 Rebalance to target at 10% drift	#7 Rebalance halfway to target at 10% drift	#8 Rebalance to target at 15% drift	#9 Rebalance halfway to target at 15% drift
Portfolio Value									
95th percentile	\$9,554,251	\$7,200,762	\$7,309,426	\$7,645,039	\$7,889,567	\$8,435,155	\$8,727,038	\$9,171,115	\$9,252,910
50th percentile	\$3,572,552	\$3,281,976	\$3,290,925	\$3,366,302	\$3,393,954	\$3,488,704	\$3,519,150	\$3,564,525	\$3,560,770
5th percentile	\$1,562,248	\$1,573,127	\$1,561,396	\$1,575,943	\$1,569,173	\$1,573,204	\$1,562,122	\$1,560,508	\$1,560,127
Maximum Drawdown									
5th percentile	\$1,813,135	\$1,108,793	\$1,140,015	\$1,200,519	\$1,258,366	\$1,414,861	\$1,506,014	\$1,593,593	\$1,671,516
50th percentile	\$701,983	\$538,589	\$545,807	\$565,568	\$584,465	\$625,367	\$649,728	\$667,284	\$685,172
95th percentile	\$335,408	\$299,460	\$303,646	\$309,697	\$314,587	\$325,140	\$329,477	\$332,066	\$333,184
Downside Risk									
> 10% decline initial 12 months	3.76%	3.76%	3.76%	3.76%	3.76%	3.76%	3.76%	3.76%	3.76%
> 15% decline any 12 months	52.34%	27.64%	28.98%	32.00%	35.30%	41.26%	44.98%	48.28%	49.56%
> 20% peak to trough decline	41.54%	20.68%	21.04%	23.18%	25.42%	31.12%	34.16%	37.32%	38.96%
Total Taxes									
5th percentile	\$893,813	\$857,034	\$885,183	\$866,032	\$897,044	\$865,241	\$897,662	\$863,603	\$892,480
50th percentile	\$508,110	\$517,202	\$528,453	\$511,763	\$522,147	\$501,700	\$511,240	\$498,929	\$507,120
95th percentile	\$326,607	\$330,754	\$335,873	\$326,622	\$331,411	\$321,857	\$326,667	\$323,539	\$325,761

Postliquidation Values (No Stepped Up Basis at Death / Distribution in cash / Noncharitable Bequest)

	#1 No Rebalance	#2 Rebalance Yearly to target	#3 Rebalance Yearly halfway to target	#4 Rebalance to target at 5% drift	#5 Rebalance halfway to target at 5% drift	#6 Rebalance to target at 10% drift	#7 Rebalance halfway to target at 10% drift	#8 Rebalance to target at 15% drift	#9 Rebalance halfway to target at 15% drift
Portfolio Value									
95th percentile	\$8,218,120	\$6,396,085	\$6,459,275	\$6,813,843	\$6,967,560	\$7,421,209	\$7,620,197	\$8,029,805	\$7,988,789
50th percentile	\$3,107,954	\$2,919,521	\$2,894,142	\$2,984,627	\$3,004,986	\$3,095,164	\$3,092,615	\$3,128,813	\$3,111,224
5th percentile	\$1,384,007	\$1,400,006	\$1,385,969	\$1,471,382	\$1,397,217	\$1,411,658	\$1,393,146	\$1,389,849	\$1,389,491
Total Taxes									
5th percentile	\$2,223,649	\$1,638,732	\$1,734,226	\$1,742,334	\$1,821,981	\$1,873,945	\$1,999,461	\$1,988,111	\$2,102,525
50th percentile	\$978,090	\$883,016	\$921,272	\$888,423	\$917,492	\$907,747	\$938,731	\$932,881	\$957,871
95th percentile	\$513,200	\$508,999	\$518,033	\$501,133	\$510,672	\$500,591	\$509,175	\$507,768	\$511,286

Summary of Portfolio Rebalance Elections under a Unitrust Distribution Policy
\$1 million starting portfolio / 70-30 allocation. End of 360 months. Values Adjusted for CPI

Preliquidation Values (Stepped Up Basis at Death / Distribution in kind / Bequest to Charity)

	#1 No Rebalance	#2 Rebalance Yearly to Target	#3 Rebalance Yearly halfway to target	#4 Rebalance to target at 5% drift	#5 Rebalance halfway to target at 5% drift	#6 Rebalance to target at 10% drift	#7 Rebalance halfway to target at 10% drift	#8 Rebalance to target at 15% drift	#9 Rebalance halfway to target at 15% drift
Portfolio Value									
95th percentile	\$2,088,095	\$1,450,302	\$1,432,744	\$1,646,848	\$1,659,639	\$1,831,474	\$1,905,561	\$1,982,066	\$2,015,174
50th percentile	\$707,312	\$615,945	\$614,335	\$677,196	\$655,687	\$700,891	\$699,738	\$708,941	\$707,817
5th percentile	\$283,881	\$282,092	\$281,102	\$297,318	\$283,944	\$293,189	\$288,064	\$287,753	\$286,235
Total Consumption									
95th percentile	\$2,158,707	\$1,847,439	\$1,857,758	\$1,976,352	\$1,989,200	\$2,084,443	\$2,110,162	\$2,126,524	\$2,142,775
50th percentile	\$1,262,479	\$1,202,167	\$1,204,888	\$1,241,707	\$1,229,902	\$1,258,037	\$1,257,980	\$1,261,447	\$1,259,820
5th percentile	\$838,919	\$829,331	\$831,764	\$841,287	\$833,905	\$840,388	\$838,759	\$839,759	\$839,437
Maximum Drawdown									
5th percentile	\$1,212,408	\$934,077	\$951,141	\$966,513	\$1,002,954	\$1,031,026	\$1,061,814	\$1,075,878	\$1,090,713
50th percentile	\$677,013	\$638,596	\$646,161	\$631,395	\$650,765	\$646,900	\$657,512	\$663,298	\$667,500
95th percentile	\$405,535	\$368,465	\$373,657	\$367,028	\$378,970	\$382,356	\$387,490	\$394,712	\$399,295
Downside Risk									
> 10% decline initial 12 months	12.00%	12.00%	12.00%	12.00%	12.00%	12.00%	12.00%	12.00%	12.00%
> 15% decline any 12 months	96.34%	84.28%	85.84%	86.20%	89.04%	91.18%	93.28%	93.86%	95.22%
> 20% peak to trough decline	97.46%	91.76%	92.78%	91.78%	93.58%	94.32%	95.30%	95.88%	96.46%
Total Taxes									
5th percentile	\$628,715	\$562,234	\$607,365	\$511,560	\$561,429	\$543,280	\$574,949	\$574,339	\$600,360
50th percentile	\$345,696	\$352,551	\$361,392	\$310,718	\$342,325	\$318,327	\$331,538	\$329,310	\$337,776
95th percentile	\$212,070	\$222,710	\$222,485	\$201,276	\$216,654	\$201,833	\$209,692	\$206,497	\$210,409

Postliquidation Values (No Stepped Up Basis at Death / Distribution in cash / Noncharitable Bequest)

	#1 No Rebalance	#2 Rebalance Yearly to Target	#3 Rebalance Yearly halfway to target	#4 Rebalance to target at 5% drift	#5 Rebalance halfway to target at 5% drift	#6 Rebalance to target at 10% drift	#7 Rebalance halfway to target at 10% drift	#8 Rebalance to target at 15% drift	#9 Rebalance halfway to target at 15% drift
Portfolio Value									
95th percentile	\$1,776,312	\$1,270,367	\$1,250,004	\$1,435,419	\$1,443,918	\$1,588,361	\$1,644,013	\$1,720,820	\$1,730,636
50th percentile	\$602,347	\$536,024	\$532,215	\$587,092	\$567,318	\$605,773	\$603,613	\$612,591	\$606,260
5th percentile	\$242,458	\$244,441	\$243,016	\$257,956	\$244,781	\$252,307	\$248,113	\$247,241	\$244,711
Total Taxes									
95th percentile	\$934,172	\$733,311	\$784,246	\$708,963	\$768,189	\$772,024	\$832,535	\$824,036	\$870,903
50th percentile	\$453,949	\$433,121	\$444,573	\$400,122	\$433,447	\$413,426	\$430,040	\$428,197	\$440,184
5th percentile	\$261,319	\$265,410	\$266,346	\$246,539	\$263,644	\$249,757	\$256,687	\$254,285	\$258,404

Summary of Portfolio Rebalance Elections under an Indexed Annuity Distribution Policy
\$1 million starting portfolio / 70-30 allocation. End of 360 months. Values Adjusted for CPI

Preliquidation Values (Stepped Up Basis at Death / Distribution in kind / Bequest to Charity)

	#1 No Rebalance	#2 Rebalance Yearly to Target	#3 Rebalance Yearly halfway to target	#4 Rebalance to target at 5% drift	#5 Rebalance halfway to target at 5% drift	#6 Rebalance to target at 10% drift	#7 Rebalance halfway to target at 10% drift	#8 Rebalance to target at 15% drift
Portfolio Value								
95th percentile	\$3,373,839	\$2,077,252	\$2,085,044	\$2,587,447	\$2,568,336	\$2,943,173	\$3,097,521	\$3,297,625
50th percentile	\$174,251	\$63,647	\$68,425	\$158,908	\$116,554	\$178,332	\$162,204	\$183,638
5th percentile	\$0	\$0	\$0	\$0	\$0	\$0	\$0	\$0
Total Consumption								
95th percentile	\$1,500,236	\$1,500,029	\$1,500,259	\$1,500,083	\$1,500,193	\$1,500,264	\$1,500,203	\$1,500,341
50th percentile	\$1,493,127	\$1,491,770	\$1,492,242	\$1,492,783	\$1,492,614	\$1,493,042	\$1,493,022	\$1,493,175
5th percentile	\$928,414	\$922,055	\$920,062	\$925,855	\$920,128	\$919,583	\$920,761	\$913,309
Maximum Drawdown								
95th percentile	\$1,283,957	\$1,207,538	\$1,221,500	\$1,207,342	\$1,220,241	\$1,239,179	\$1,241,290	\$1,250,785
50th percentile	\$1,000,000	\$1,000,000	\$1,000,000	\$1,000,000	\$1,000,000	\$1,000,000	\$1,000,000	\$1,000,000
5th percentile	\$453,861	\$406,929	\$403,087	\$398,271	\$409,807	\$411,461	\$427,525	\$426,945
Downside Risk								
% trials bankrupt	42.14%	46.90%	46.26%	43.34%	44.58%	41.72%	42.62%	42.10%
% trials < \$200,000	45.44%	50.92%	50.82%	46.88%	48.56%	45.66%	46.36%	45.48%
> 10% decline initial 12 months*	11.22%	11.48%	11.40%	11.00%	10.96%	11.34%	10.76%	10.64%
> 15% decline any 12 months	95.88%	88.32%	88.94%	87.60%	90.50%	90.74%	92.70%	93.46%
> 20% peak to trough decline	95.16%	90.22%	89.84%	89.00%	90.84%	90.76%	92.24%	92.82%
Total Taxes								
5th percentile	\$600,981	\$588,906	\$642,475	\$525,084	\$569,044	\$542,775	\$567,701	\$559,538
50th percentile	\$352,990	\$351,389	\$357,333	\$317,365	\$348,605	\$325,173	\$337,991	\$337,114
95th percentile	\$169,095	\$168,673	\$168,325	\$162,932	\$168,183	\$164,072	\$166,273	\$166,316
End Month: 5th percentile	224	222	222	223	222	221	222	220

Postliquidation Values (No Stepped Up Basis at Death / Distribution in cash / Noncharitable Bequest)

	#1 No Rebalance	#2 Rebalance Yearly to Target	#3 Rebalance Yearly halfway to target	#4 Rebalance to target at 5% drift	#5 Rebalance halfway to target at 5% drift	#6 Rebalance to target at 10% drift	#7 Rebalance halfway to target at 10% drift	#8 Rebalance to target at 15% drift
Portfolio Value								
95th percentile	\$2,870,008	\$1,830,033	\$1,819,965	\$2,276,184	\$2,243,142	\$2,577,449	\$2,690,163	\$2,852,957
50th percentile	\$148,179	\$54,094	\$58,153	\$136,202	\$99,306	\$152,239	\$138,662	\$158,960
5th percentile	\$0	\$0	\$0	\$0	\$0	\$0	\$0	\$0
Total Taxes								
95th percentile	\$1,096,964	\$832,765	\$905,176	\$833,590	\$892,088	\$908,372	\$974,318	\$984,207
50th percentile	\$380,521	\$363,251	\$369,027	\$343,023	\$367,750	\$353,248	\$363,748	\$364,010
25th percentile	\$169,095	\$168,673	\$168,325	\$162,932	\$168,183	\$164,072	\$166,273	\$166,316

*Failure rates differ per tax, asset management, and investment expense fees cumulated throughout the year. Simulations absent these portfolio frictions would produce the same initial failure rate across all rebalance elections