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MODELING FUTURE INVESTMENT RESULTS: PORTFOLIO ANALYTICS

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PREFACE

Portfolio Analytics provides technical information on the portfolio model used in the Investment Policy Statement [IPS]. The model employs historical returns, variances and asset class correlations for the period January 1973 through the end the previous year. The model focuses on the economic consequences of the strategic asset allocation decision. It does not take into account contributions and withdrawals, expenses, and taxes. As such, it is an asset allocation model not a prediction of actual future wealth or portfolio distributions (consumption). The model's output is:

1. Constant (inflation-adjusted) dollar wealth; and,
2. Real (inflation-adjusted) investment return.

There are many ways to model a portfolio's evolution as determined by its asset allocation. The dollar values generated by one model may differ, sometimes substantially, from those generated by another. This phenomenon is termed "model risk" to distinguish it from "investment risk"—the risk of uncertain future returns and inflation. Model risk increases to the extent that a model's output is sensitive to input assumptions. Clearly, a model that projects only average historical returns into all future years is not credible although it meets tests for logical consistency. Likewise, a model that assumes financial returns are normally distributed will miss the extreme results that characterize historical returns.

Historical return series are "messy." They exhibit serial correlation, dynamic correlation, time-varying volatility, return asymmetries (skewness), and a propensity for "outliers" (kurtosis). Models, however, are "neater." They attempt to express a functioning system with mathematical equations. Although a good model will capture much of these statistical behaviors, no model can hope to replicate reality. Therefore, our model is deliberately conservative in the sense that it generates outputs that do not assume constant parameters.

A conservative model, in this context, is one that generates a positive probability for severe downside returns—comparable to the Great Depression, or worse. Any model can deliberately slant inputs to generate "worst case" returns. Model builders can stress test a portfolio by deliberately manipulating model inputs. However, a more credible approach is to develop a model that generates substantial downside risk without predetermining input values just to arrive at a targeted number. It is difficult enough to predict the weather—it is next to impossible to predict a confluence of events that will cause the "perfect storm."

This document presents details for a Regime-Switching, Markov-Transition model which simulates a broad range of returns over various future economies. The model inputs historical returns in favor of forecasted returns in order to achieve unbiased outputs. However, historical returns during the period 1973 through the present suggest that certain asset classes generate systematically better risk-adjusted returns relative to the broad market. For example, many argue that small stocks and value stocks should provide investors with a risk-premium relative to large stocks and growth stocks. If realized returns reflect all underlying risk factors, then historical returns are a reasonable model inputs. If return inputs are modified to conform to the traditional Capital Asset Pricing Model, then small and value stocks may not provide the risk/return benefits expected by the investor. This is yet another aspect of model risk.

THE NATURE OF THE INVESTMENT PROCESS

Investing is a risky activity because it is impossible to predict future outcomes precisely. It is possible, however, to estimate the level of risk assumed by a portfolio through an examination of historical return variability. The portfolio model of future expected returns attempts to reflect periods of distress—bear markets-—by including data from the 1973-1974 oil-crisis era, the Reagan and Bush One recessions, the Asian Banking Crisis, the NASDAQ market meltdown, terrorist attacks, as well as the recent global recession. It also reflects periods of prosperity—bull markets—from 1973 through the present.

How an individual investor reacts to periods of asset price advances and declines depends primarily on two factors: (1) the ratio of investment wealth to consumption from sources other than labor income or entitlements, and (2) personal risk aversion. If current wealth is large relative to the income demands made against it then, during periods of extreme investment decline, the risk of investing becomes the risk of overreaction—that is, a desire to sell. During periods of extreme investment upturn, the risk of investing becomes the risk of trend following—that is, jumping on the buy-side bandwagon. However, if current wealth is small relative to the income demands made against it, then the investor must be wary of further losses lest they permanently impair the portfolio's ability to meet crucial objectives.

The portfolio model presented in your IPS is an important first step in developing a prudent investment process because it sets a strategic asset allocation appropriate for your goals and current circumstances. It is, however, the beginning of an ongoing asset management process—not the end. *Portfolio Analytics* focuses primarily on the assumptions underlying the strategic asset allocation model. The topics of portfolio supervision and monitoring lie beyond this discussion although they, too, are critically important for long-term investment success.

The IPS model should be taken as a long-term risk/reward model rather than as a market forecast or prediction. The model measures inflation-adjusted change in wealth per $1,000 of initial investment, and it does not represent any particular investment portfolio. It focuses only on the economic consequences of the strategic asset allocation decision because it does not incorporate the effects of taxes, expenses, contributions or withdrawals. Like all models, it generalizes and abstracts the full complexity and detail of the environment that it mirrors.

Actual future portfolio returns unfold in a process that moves through time and that is subject to uncertainty. This means that forecasting future results is not an exercise in calculating a single "expected value" for a portfolio. Rather, the profile of any portfolio, as it moves through time, involves a statistical spread (probability distribution) of possible future returns. We can never know the probability distribution with exact certainty nor can we know the outcome along any path because of the changing period-to-period future transition probabilities.[1] Complicating matters further, we have only a single sample of historical prices upon which to build the model. As Nobel Prize winning economist Paul

[1] For example, the probability of transitioning from a low inflationary environment to a high inflationary environment; or, the probability of a change in tax law regimes.

Samuelson reminds us: "Only a fool makes predictions based on a sample of one." The insights granted to us when we build models should, therefore, not be confused with actual asset pricing forecasts.

The mid-point within the range of future values is the "median value." Median value separates the top 50% of real return results from the bottom 50% of real return results. Thus, in one sense, median value is the statistically expected return. Expected return, paradoxically, is never to be expected. Although the expected return of a single roll of one die equals 3.5 (each number, 1 through 6, on the die has a one/sixth chance of appearing; the sum of the numbers divided by the probability of their occurrence equals 3.5), you will never actually see this result. In a continuous time process, the chance of achieving any particular estimated or forecasted value is, likewise, close to zero.

If a hypothetical investor owned a portfolio with a given strategic asset allocation, and if the portfolio could be invested over thousands of independent twenty-year periods, then the model suggests that most results should cluster towards the median value. However, in some of the twenty-year periods, results may lie far away from the median. The model suggests what might happen over many twenty-year trials; it cannot predict what will happen in a single twenty-year trial. Thus, prudent investing under conditions of uncertainty requires not only that you implement a portfolio that enhances the chances for long-term investment success; but, also, that you monitor results to determine if it remains on track.

MODELING RISK: SIMULATION OF A MARKOV REGIME SWITCHING MODEL

Simulation is an approach to modeling that seeks to mimic a functioning system as it evolves. It is built on mathematical equations that express the assumed form of the system's operation. Simulation models assume a range of complexity from (1) a simple "bootstrap" of time series data in which periodic returns are sampled with replacement to create a large number of reshuffled return sequences; to, (2) a simple structural model like Monte Carlo simulation which draws random samples from a pre-specified distribution; to (3) more complex simulation models that blend distributions according to certain probability criteria.

The IPS simulation model incorporates several "moving parts" which are best characterized as random variables. These include:

The Planning Horizon—the applicable planning horizon can either be fixed (e.g., a university endowment will need to finance a new building in exactly 7 years) or variable (an indeterminate length). The IPS model illustrates the simulated range of investment results after 1, 3, 5, 10 and 20 years.

The Economy—the IPS model divides future economies into two regimes: A Bear Market regime (defined as a 20% or greater peak to trough price decline for the Capital Appreciation S&P 500 stock index); and, a Bull Market regime. Using historical data from January 1973 through the end of last year, the historical lengths of bull and bear markets are determined. The simulation uses a Markov-switching process (with a random selection for the initial economic regime) to determine the sequence of market conditions that will be faced by the investor. The probability (p) that the initial economy is in a bull market regime or a bear market regime (1 – p) is based on historical frequencies. For all future periods, the simulation determines the probability of remaining in a bear market given that the last month was a bear; or, of switching from a bear to a bull market given the total duration of the bear market to date. Similar calculations are made for the probability of remaining in or leaving a bull market regime.

Inflation—the model proxies inflation by the Consumer Price Index. Changes in the inflation rate are primary determinates of the likelihood that periodic investment returns are either positive or negative. The model specifies the inflation generating process as a serially correlated random variable with a "smoothed" reversionary factor. Specifically, the algorithm regresses the average value of the previous 12 month's inflation against the average value of the next twelve months. The value is calculated as:

Inflation_t = long term inflation + Persistence Coefficient [Sum(inflation_{t-1} … inflation_{t-12}) / 12 – long term inflation] + error term

Where the error term is an iid, "white noise" process.

When the application has not yet produced twelve monthly simulated values, the application recursively calculates the average of the preceding twelve months by using the initial value to replace any missing terms. Therefore, the value for average prior twelve month inflation in the second month is 11/12 * the initial value + 1/12 * the value in the first month.

The persistence Coefficient determines the speed of CPI mean reversion. The coefficient's value is calculated via a regression of the rolling twelve-month CPI against the rolling forward twelve-month CPI. Thus the model assumes that inflation is an Ornstein-Uhlenbeck process which includes a term for autocorrelation as well as for mean-reversion. All outputs are discounted back to present value by the inflation rate.

Investment Returns—the simulation model generates investment returns using common matrix algebra techniques. Utilizing separate variance/covariance matrices from historical bull and bear market regimes, the application executes a Cholesky decomposition. It may also adjust dependence relationships by shrinking extreme off-diagonal elements to assure matrix invertibility. The Cholesky matrix algebra operation "divides" a variance/covariance matrix into upper and lower triangle matrices which make them equivalent to the square root of a variance matrix. If there exists a lower triangle matrix **C** such that the historical matrix $\mathbf{V} = \mathbf{CC}^{\mathbf{t}}$, then **C** is a Cholesky matrix. The application simulates combinations of return series where each historical return series $(\vec{x})$ is transformed (by subtracting the mean and dividing by the standard deviation) into an independent standard normal variable $(\vec{z})$.

The computer's random number generation function can readily simulate future evolutions for each independent return vector by drawing values for uncorrelated zero-mean variables. Pre-multiplying the vectors of simulated independent returns by **C** $C(\vec{z})$ restores their equivalency to each original return series $(\vec{x})$ = $C(\vec{z})$. The variance of the independent vectors is easily determined; and, pre and post multiplication of the variance of $(\vec{z})$ by the appropriate lower triangle decomposition matrix **C** and its inverse restores the correlation structure by generating the required variance/covariance matrix. [**V** = **C**V$(\vec{z})$**C**^{t}]

Financial asset return series usually cannot be characterized as normal (bell-curve) distributions. Portfolio investment risk defined by the first two moments of multivariate symmetric distributions (Gaussian, Student's t, Cauchy, etc.) is often misleading. Monte Carlo simulations based on a normal distribution cannot realistically capture the frequency and magnitude of tail-risk events (leptokurtosis). To avoid this deficiency, the application utilizes two normal distributions (bull and bear) with separately calculated means and variances for each regime. The distributions, according to the Markov transition probabilities described above, enable the model to capture the risk of outlier results that mirror real world frequencies rather than risks that are largely predetermined by theoretical parameter inputs.

Additionally, a regime switching approach captures dynamic correlation and time-varying risk premia over different market conditions. Thus, instead of using average unconditional correlation values determined by the historical data, the application applies the historical correlation values conditioned on bull and bear market data. For example, over the entire sample period, an asset class may exhibit a mean of 10% and a standard deviation of 20%. However, during bull markets, the parameter values may be +18% mean and 15% standard deviation; while, during bear markets, the parameter values by be -23% and 25% respectively. Thus, simply using the unconditional mean, standard deviation and correlation values for the aggregate historical period cannot capture realistic asset price behavior.

The IPS model inputs the above random variables to drive portfolio evolutions over a wide range of possible future economies. Given the large number of simulation paths (5,000 trials), there is a rich set of future asset returns. It should be recognized, however, that any model is an imperfect representation of a more complex reality. In this case, there are (at least) two "model risks" which should be considered:

1. Investors are interested in forecasts of a price change process. However, the time series of asset prices is not statistically 'stationary' (i.e., exhibits the potential for infinite variance). It is only by "differentiating" the logarithm of prices on a period-by-period basis that the creation of a stationary series of returns is possible. That is to say, it is only possible to model asset *returns*; but an investor measures wealth based on asset *prices*. This is a subtle but important distinction. Returns are based on the single historical path of price changes, the realization of which is merely a manifestation of an unknown price generating process. Simulation analysis greatly broadens our perspective about possible future outcomes; but any model of such a process must remain only a crude approximation of reality. Indeed, calculation of investment return is a function of the measurement interval (yearly, monthly, daily, intraday, continuous time) and, at the limit, may be meaningless in a statistical context.

2. The single historically realized return path for each asset class may be "representative" of the unknown price generating process; or, may merely be an outlier result unlikely to persist. For example, an asset allocation tilt towards small and value stocks is based on historical return data. If the premium for investing in small and value stocks reflects a reward for systematic risks, then investors have some confidence that they will continue to be rewarded for making these investments. If, however, the premium for such investments is merely an artifact of a chance historical price process, then investors may be increasing risk as they move their asset allocation deeper into the small/value gradient. Furthermore, investment volatility is measured by the variance statistic [the squared difference between actual returns and average return]. But if the historical return path is not representative, then the concept of average becomes meaningless and statistical measures are not illuminating.

Investors are rewarded for taking prudent and calculated risks. Investors may use the historical data to make inferences concerning the interrelationships between asset allocation, risk and reward. However, in designing and implementing a portfolio, it is always wise to remain aware of parameter uncertainties. Past performance is not a guarantee of future results.

DATA SOURCES

PORTFOLIO ASSET CLASS	PROXY BENCHMARK	DESCRIPTION OF RETURN SERIES
EQUITY		
US Large Company Stocks	S&P 500 Stock Index	1/73 to present: S&P 500 Index reported in Ibbotson & Sinquefield Stocks, Bonds, Bills and Inflation
US Large Company Growth Stocks	CRSP US Large Cap Growth Return Series Index	1/73 to present: Fama-French US Large Cap Growth Index from Center For Research in Securities Prices (CRSP) data base
US Large Company Value Stocks	CRSP US Large Cap Value Return Series Index	1/73 to present: Fama-French US Large Cap Value Index from Center For Research in Securities Prices (CRSP) data base
US Mid Cap Stocks	MSCI US Mid Cap 450 Index	1/92 to present: Morgan Stanley Capital International US Mid Size Companies
US Mid Cap Growth Stocks	MSCI US Mid Cap Growth Index	1/92 to present: Morgan Stanley Capital International US Mid Growth Companies
US Mid Cap Value Stocks	MSCI US Mid Cap Value Index	1/92 to present: Morgan Stanley Capital International US Mid Value Companies
US Small Company Stocks	CRSP 6-7-8 Deciles Stock Return Series Index	1/73 to present: CRSP New York, American and NASDAQ Market Total Return Index
US Small Company Growth Stocks	CRSP US Small Cap Growth Return Series Index	1/73 to present: Fama-French US Small Cap Growth Index from Center For Research in Securities Prices (CRSP) data base
US Small Company Value Stocks	CRSP US Small Cap Value Return Series Index	1/73 to present: Fama French US Small Cap Value Index from CRSP data base
US Micro Cap Stocks	CRSP 9-10 Deciles Stock Return Series Index	1/73 to present: CRSP New York, American and NASDAQ Market Total Return Index
International Large Company Stocks	Morgan Stanley Capital International EAFE (Europe, Australia, Far East) Index	1/73 to present: MSCI EAFE Index
International Large Company Value Stocks	MSCI EAFE Value Index	1/75 to present: MSCI EAFE Value Index
International Small Company Stocks	DFA International Small Company Return Series	Allocations based on DFA International Small Stock Trust through 1994, and on DFA International Small Stock Portfolio thereafter:
International Small Company Value Stocks	S&P EPAC Small Value index	1/89 to present: S&P Europe, Pacific, and Asia Composite Index of Small Value companies
Emerging Markets Stock	International Finance Corporation Emerging Markets Investable Composite Total Return Series	1/89 to present: The IFCI Index represents the performance of stocks in emerging markets that are available to foreign institutional investors.
Real Estate	NAREIT (National Association of Real Estate Investment Trusts) Equity Return Series	1/73 to present: REITS with 75% or greater of their gross invested book assets invested directly or indirectly in equity ownership of real estate. Index is a market valued weighted index based upon the last closing price of the month for tax-qualified REITs listed on the NYSE, AMEX and the NASDAQ.

FIXED INCOME		
Short Term Taxable Fixed Income	One Year Constant Maturity US Treasuries	1/73 to present: US Treasury One-Year Constant Maturity Bond Total Return extrapolated from Federal Reserve Board of Governors' data.
Intermediate Term Taxable Fixed Income	Barclays Capital Intermediate Term Government / Corporate Bond Index	1/73 to present: BC Government / Corporate Bond Index with maturities up to 10 years.
Long Term Taxable Fixed Income	Barclays Capital Long Term Government / Corporate Bond Index	1/73 to present: BC Government / Corporate Bond Index with maturities up to 30 years.
Intermediate Term Global Income	Citigroup 1+ Year World Gov't Bonds	1/85 to present: Citigroup Global Bond Index. Unhedged and denominated in US Dollars.
Treasury Inflation Protected Securities	Barclays Capital Global Inflation Linked US TIPS	1/97 to present: Barclays Capital Global Inflation Linked US Treasury Inflation Protected Securities
Aggregate Bond Market	Barclays Capital US Aggregate Bond Market Index	1/76 to present: Barclays Capital US Aggregate Bond Market Index of private and public debt
US High Yield Bond	Barclays Capital US Corporate High Yield Index	1/73 to present: Barclays Capital US Corporate High Yield Bond Index
US Mortgage Backed Securities	Barclays Capital US Mortgage Backed Securities Index	1/76 to present: Barclays Capital US Index of Mortgage Backed Securities
Muni National Short Term Bonds	Barclays Capital Municipal 3 Year Bond Index	1/90 to present: Total Return index for the investment-grade tax-exempt national bond market. Index has an average duration of 3 years. General Obligation, Revenue, Insured, and Pre-refunded Bond sectors included.
Muni National Intermediate Term Bonds	Barclays Capital Municipal 7 Year GO Bond Index	2/85 to present: Barclays Capital US national municipal bond index with average duration of 7 years and investments only in general obligation bonds
Muni National Long Term Bonds	Barclays Capital Municipal 10 Year GO Bond Index	2/80 to present: Barclays Capital US national municipal bond index with average duration of 10 years and investments only in general obligation bonds
California Muni Bonds	Barclays Capital Municipal California Exempt	1/93 to present: Total Return series for California municipal bond index with a minimum credit rating of BAA3, a maturity of 1 year or greater, and have been issued after December 31, 1990.
OTHER		
Commodities	S&P GSCI	1/70 to present: Formerly the Goldman Sachs Commodities Index. The index has a high allocation towards energy. Commodities are categorized as Fixed Income for the purposes of this document because the affect of taxes on Commodities funds are more similar to Fixed Income funds than Equity funds.